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PAPER

GENERAL

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Bayesian Networks and the Value of the Evidence for the Forensic Two-Trace Transfer Problem*

ABSTRACT: Forensic scientists face increasingly complex inference problems for evaluating likelihood ratios (LRs) for an appropriate pair of propositions. Up to now, scientists and statisticians have derived LR formulae using an algebraic approach. However, this approach reaches its limits when addressing cases with an increasing number of variables and dependence relationships between these variables. In this study, we suggest using a graphical approach, based on the construction of Bayesian networks (BNs). We first construct a BN that captures the problem, and then deduce the expression for calculating the LR from this model to compare it with existing LR formulae. We illustrate this idea by applying it to the evaluation of an activity level LR in the context of the two-trace transfer problem. Our approach allows us to relax assumptions made in previous LR developments, produce a new LR formula for the two-trace transfer problem and generalize this scenario to n traces.

KEYWORDS: forensic science, evaluation of transfer evidence, Bayesian networks, two-trace problem, likelihood ratio, graphical probability models, object-oriented Bayesian networks

A dead body is found in a public park. The medical examination of the body reveals signs of a physical struggle. On the crime scene, forensic investigators recover two items of a certain category of trace evidence, say, for example, bloodstains. Trace 1 was found in what we will call location 1 and trace 2 in location 2. These are both inside the perimeter of the crime scene.

The investigation is able to acquire images from two surveillance cameras which filmed different parts of the crime scene during the time lapse the crime was committed. One camera filmed location 1 and the other location 2. Each camera has an image showing the victim struggling with an assailant: the image from the first camera shows the victim and an assailant in location 1, the image from the second camera shows the victim and the assailant in location 2. Unfortunately, the images are of poor quality, and a comparison does not allow one to conclude with certainty whether it is the same assailant in both locations, or whether there were two assailants, one in location 1 and the other in location 2.

A forensic laboratory analyzes an intrinsic characteristic of the two traces (e.g., the blood group or DNA profile). A comparison of these analytical results with the result obtained from the victim's sample allows the forensic scientists to exclude the victim as the source of both of these traces. The results show that the traces are of different types, say, for example, that trace 1 is of type Γ_1 and trace 2 of type Γ_2.

Later in the investigation, a suspect's sample matches one of these two traces. The evidence against this suspect thus consists of a combination of a matching item and a nonmatching item. The question is, "How strong is this evidence against the suspect?"

In forensic science, this scenario is known as the two-trace transfer problem. The answer to the above question takes the form of a likelihood ratio (LR). This LR opposes two mutually exclusive propositions (where the word *proposition* designates a formal statement about an event that the forensic scientist formulates based on the circumstantial information of the case, as described by [1]):

$$\mathrm{LR} = \frac{\Pr(\text{evidence}|\text{proposition } 1, I)}{\Pr(\text{evidence}|\text{proposition } 2, I)}$$

The LR is the ratio of the probabilities of observing the evidence given each of the competing propositions and I. The evidence consists of the characteristics of the two traces and the suspect's sample, and the letter I denotes the background information. The background information consists of all the knowledge and circumstances that influence the numerical evaluations of the probabilities forming the numerator and the denominator. Note that all probabilities in this study are conditional on the background information I, yet, for the sake of brevity, we shall hereafter omit I from their notation.

For this scenario, the forensic scientists can formulate the following pair of source level propositions (we use the superscript s to indicate that these are source level propositions):

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H^s—*one of the traces on the crime scene comes from the suspect;*
$\bar{H}^s$—*neither of the traces on the crime scene comes from the suspect;*

or the following pair of activity level propositions (we use the superscript a to indicate that these are activity level propositions):

H^a—*the suspect was engaged in a struggle with the victim;*

$\bar{H}^a$—*the suspect was not engaged in a struggle with the victim.*

The first pair of propositions is at the source level in the hierarchy of propositions (2), because it is only concerned with the origin of the traces, that is, the item or person from which the traces come from. This pair ignores *how* and *when* the traces were transferred from their origin to the crime scene. A match between the profile of the suspect's sample and one of the traces, say for example trace 1, produces an LR supporting proposition H^s, regardless of the extrinsic characteristics of the traces (e.g., the quantity of material on the crime scene, or how fresh the trace was on the crime scene). If the trace is an old bloodstain, that was already present on the crime scene before the victim was assaulted, then this evidence will support H^s with an LR equal to $\frac{1}{2\gamma_1}$ (3), just as in the case where the bloodstain was transferred to the crime scene during the assault. This LR only takes into account γ_1, which denotes the probability of obtaining a match with trace 1's analytical characteristic Γ_1 (the matching characteristic, in this case) in the relevant population (4). For this reason, the LR for this pair of propositions only provides us with a very limited amount of information, which does not tell us whether the suspect was an assailant who struggled with the victim. To address the question of whether the suspect was an assailant, we must use the second pair of propositions.

The second pair of propositions is formulated at the activity level in the hierarchy of propositions (2), because it describes the activity or action of interest to the case, that is, the activity or action that may have caused the transfer of the traces from the assailant to the crime scene. In addition to taking into account the origin of the traces, these propositions also consider *how* and *when* the traces came onto the crime scene. An activity level LR thus consists of transfer, background, and match probabilities (5). To differentiate the probabilities referring to trace 1 from those referring to trace 2, we use the subscript $i \in \{1, 2\}$ for all of the probabilities referring to trace i and location i:

- *Transfer probabilities* t_i, i = 1, 2, describe how probable it is for trace i to have been transferred during the alleged action, have persisted on the crime scene, and then to have been recovered by the investigators. The complement of t_i is $\bar{t}_i = 1 - t_i$.
- *Background probabilities* b_i, i = 1, 2, represent the probability for a trace to be present on the crime scene at location i as a consequence of another transfer event, unrelated to the alleged action. The probability of the absence of such a trace is $\bar{b}_i = 1 - b_i$.
- *Match probabilities* γ_i, i = 1, 2, are the probabilities for obtaining a match with characteristic Γ_i (in our scenario, we call trace 1's characteristic Γ_1 and trace 2's characteristic Γ_2, such that the subscript i here corresponds with the notation for trace i, $i \in \{1, 2\}$) in the relevant population. For a trace transferred during the struggle, the relevant population is that of the possible assailants. For a background trace, however, the relevant population is the population of background traces. To distinguish these two from each other, the match probability of a characteristic in the population of possible assailants is γ_i, and the match probability of a characteristic in the population of background traces is denoted γ'_i.

Thus, we have, for example, $\bar{b}_1$ denoting the probability that there was no background trace present at location 1, and t_2 denoting the probability that a trace was transferred to location 2 during the struggle between the assailant and the victim at that location, persisted there, and was recovered during the investigation. For the sake of simplicity, we will assume in the rest of the paper that the traces transferred during the alleged action all persisted on the crime scene and were all recovered by the investigators. Therefore, we will simply refer to these traces as traces that were transferred during the struggle.

Using an algebraic approach, previous authors (6) came up with the following formula for evaluating this activity level LR:

$$\mathrm{LR} = \frac{\begin{array}{c}\frac{1}{2}\mathrm{b}_1\mathrm{b}_2\mathrm{t}_1\mathrm{t}_2(1-2\mathrm{q})\gamma_2 + \frac{1}{2}\mathrm{b}_1\mathrm{b}_2\mathrm{t}_1\mathrm{t}_2(1+\gamma_1)\gamma'_2 \\ +\frac{1}{2}\mathrm{b}_1\mathrm{b}_2\mathrm{t}_1\mathrm{t}_2\gamma'_1\gamma_2 + \mathrm{b}_1\mathrm{b}_2\mathrm{t}_1\mathrm{t}_2\gamma'_1\gamma'_2\end{array}}{\begin{array}{c}\mathrm{b}_1\mathrm{b}_2\mathrm{t}_1\mathrm{t}_2(1-2\mathrm{q})\gamma_1\gamma_2 + \mathrm{b}_1\mathrm{b}_2\mathrm{t}_1\mathrm{t}_2\gamma_1\gamma'_2 \\ + \mathrm{b}_1\mathrm{b}_2\mathrm{t}_1\mathrm{t}_2\gamma'_1\gamma_2 + \mathrm{b}_1\mathrm{b}_2\mathrm{t}_1\mathrm{t}_2\gamma'_1\gamma'_2\end{array}} \tag{1}$$

This equation combines the above described transfer, background, and match probabilities. In addition, it contains the expression 1 − 2q to describe the probability that the transferred traces come from two different assailants (6). That is, these authors assumed that there were two assailants and defined the probability that two transferred traces both come from assailant 1 as q, and the probability that two transferred traces both come from assailant 2 as q. This led to a probability of 2q that two transferred traces come from the same assailant, and to a probability of 1 − 2q that two transferred traces come from different assailants.

As for the background probabilities, note that in this previous study (6), a single variable p was used to describe the background probabilities of both traces together, such that $\bar{b}_1 \times \bar{b}_2 = p_0$, $\bar{b}_1 \times b_2 = p_1^2$, $b_1 \times \bar{b}_2 = p_1^1$, and $b_1 \times b_2 = p_2^{1,2}$. Here, a b is used instead of p to be able to compare the formula with other activity level formulae figuring in later sections.

The authors of (6) developed Eq. (1) using an algebraic approach. That is, they considered four mutually exclusive transfer events to explain the evidence: the product of an appropriate combination of the transfer, background, and match probabilities depicts each of these transfer events, and the sum of these four products (one for each possible event) forms the numerator and the denominator of this LR.

Aim and Outline of this Study

Despite the valuable formal rigor of such an approach, it reaches its limits when applied to increasingly complex inference problems: either it will make simplifying assumptions that ignore probabilistic dependencies between the variables, or the mathematical development of the formula becomes so intricate that it is no longer transparent to nonstatisticians (such as lawyers, prosecutors, and judges). With regard to this issue, the aim of this study is to investigate a new approach for developing an LR formula for complex forensic inference problems: a graphical approach based on the construction of Bayesian networks (BNs). BNs have already proven to be practical tools for portraying inference problems in forensic science (e.g., [7–17] to name a few). Yet, up to now, forensic statisticians have used BNs to reproduce existing LR formulae, formulae developed through algebraic calculations. Instead of coming up with an algebraic formula, and then translating it into a BN, our approach inverses this process: first, we will construct a new BN that captures the problem by combining existing BNs in a logical way, and then, in a second step, verify the logic behind this BN by analyzing the mathematical expression for computing the LR produced by this network.

We will demonstrate the potential of this approach by applying it to the two-trace transfer problem described above. Up to now, only (6) have proposed a formula for evaluating the corresponding LR (Eq. [1]) at the activity level. Note that our approach will not follow the reasoning that led to the development of Eq. (1). In this study, we relax the prior assumption of their being two assailants, as well as the prior assumption of it being equally probable for the suspect to have been the assailant of the victim in each of the two locations in the case of two different assailants.

This paper is organized as follows. First, we explain what BNs are and describe the BN for evaluating an activity level LR for a scenario involving the recovery of only a single trace. Second, we extend this reasoning process to the recovery of two traces in the two-trace transfer problem by constructing a new BN. Following this result, we deduce from the constructed network the algebraic expression corresponding to the model's computed LR, a formula, which we then compare with Eq. (1) and discuss in different situations, including an extension to n traces.

Bayesian Networks

A BN (also known as a probabilistic expert system) is a directed acyclic graph composed of nodes and arrows (18,19). Nodes stand for random variables that can be either discrete or continuous—for the sake of simplicity, the examples in this study will all use discrete nodes. Hence, each variable will consist of a finite number of exhaustive and mutually exclusive states. The arrows represent probabilistic relationships between the variables. Each arrow connects a *parent* node to a *child* node and conditions the probability distribution of the child node upon its parent. Probability tables allow the user to quantify these probabilistic relationships. For an explanation of the different categories of relationships between variables that may be modeled by a BN, see for example, (20).

The key advantage offered by BNs is their capacity of splitting up a complex inference problem into its different variables. In this way, a BN decomposes the joint probability distribution of a set of random variables $X_1,\ldots,X_n$ into the product of their probabilities conditioned on their parents, which is nothing else than the Markov property:

$$\Pr(X_1,\ldots,X_n)=\prod_{i=1}^{n}\Pr(X_i|\text{parents}(X_i))$$

Note that several different BN structures may be accepted as a description of the same scenario. There is no true model; a model is personal and reflects the constructor's view of the problem and the information available at the time of its construction (21). Thus, as our understanding of the issue progresses, a constructed network may evolve to model a situation more accurately.

For constructing the BNs in this study, we used the software *Hugin Researcher 6.7*, by Hugin Expert A/S (DK-9000 Aalborg, Denmark).

Example of a Source Level Bayesian Network for a Single Trace

To illustrate the use of BNs, Fig. 1 shows a BN for evaluating the source level LR for a single trace. This network consists of three variables:

- F for the pair of propositions:

F^s—the trace on the crime scene comes from the suspect;
$\bar{F}^s$—the trace on the crime scene does not come from the suspect;

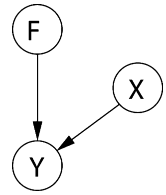

FIG. 1—*BN for evaluating a source level LR for a single trace. Node* F *contains the pair of propositions F^s and $\bar{F}^s$, and nodes* X *and* Y *contain an exhaustive list of the possible analytical results of the analyses of the suspect's sample (node* X*) and of the trace recovered on the crime scene (node* Y*). Table 1 gives the conditional probability table for node* Y.

(we use the capital letter F to distinguish the propositions in a one-trace problem from the propositions in a multiple trace problem denoted with the capital letter H),

- X for the characteristic of the suspect's sample, and
- Y for the characteristic of the trace recovered on the crime scene.

The states of nodes X and Y are an exhaustive list of the possible analytical results of the laboratory analysis (e.g., the possible blood groups or genotypes of a DNA marker). For the sake of illustration, consider that these possible results are limited to three: Γ_1, Γ_2, and Γ_{other}, where Γ_{other} groups together all of the possible analytical results that are neither Γ_1, nor Γ_2.

The relationship between the three variables is the following: if F^s is true, then the characteristic of the trace must be the same as the characteristic of the suspect's sample; and if $\bar{F}^s$ is true, the characteristic of the trace is assumed to be independent of the characteristic of the suspect's sample. Note that the BNs in this study do not include the possibility of laboratory errors (to introduce this possibility into a BN, see e.g., [12,22]). The characteristic of the trace, therefore, depends on which proposition is true and on the characteristic of the suspect's sample. This makes node Y a child of nodes F and X (Fig. 1). Table 1 presents the conditional probability table associated with node Y.

A match between the recovered trace and the suspect's sample—say, for example, $Y=\Gamma_1$ and $X=\Gamma_1$—produces the following LR for propositions F^s and $\bar{F}^s$:

$$\text{LR}=\frac{\Pr(Y=\Gamma_1,X=\Gamma_1|F^s)}{\Pr(Y=\Gamma_1,X=\Gamma_1|\bar{F}^s)}$$

Applying the third law of probability for dependent events (23,24) produces:

$$\text{LR}=\frac{\Pr(Y=\Gamma_1|X=\Gamma_1,F^s)}{\Pr(Y=\Gamma_1|X=\Gamma_1,\bar{F}^s)}\times\frac{\Pr(X=\Gamma_1|F^s)}{\Pr(X=\Gamma_1|\bar{F}^s)}$$

The characteristic of the suspect's sample ($X=\Gamma_1$) is independent of the propositions, such that $\Pr(X=\Gamma_1|F^s)=$

TABLE 1—*Probability table for node* Y *in Fig. 1. For simplicity, we use only three categories to describe the analytical results: Γ_1, Γ_2, and Γ_{other} (where Γ_{other} groups together all of the possible analytical results that are neither Γ_1, nor Γ_2).*

F:	F^s			$\bar{F}^s$		
X:	Γ_1	Γ_2	Γ_{other}	Γ_1	Γ_2	Γ_{other}
Y:						
Γ_1	1	0	0	γ_1	γ_1	γ_1
Γ_2	0	1	0	γ_2	γ_2	γ_2
Γ_{other}	0	0	1	$1-\gamma_1-\gamma_2$	$1-\gamma_1-\gamma_2$	$1-\gamma_1-\gamma_2$

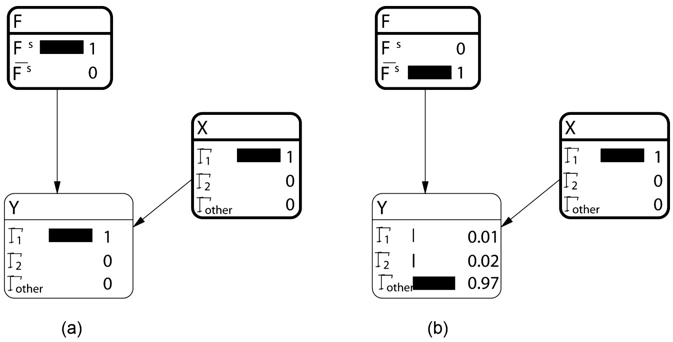

FIG. 2—*The BN in Fig. 1 computes the probabilities forming the LR (Eq. 2). The bold contour indicates that the node is instantiated. Here γ_1 = 0.01 and γ_2 = 0.02. (a) The numerator of the LR is the probability of $Y = \Gamma_1$ when states $X = \Gamma_1$ and F^s are instantiated; (b) the denominator the probability of $Y = \Gamma_1$ when states $X = \Gamma_1$ and $\bar{F}^s$ are instantiated.*

$\Pr(X = \Gamma_1|\bar{F}^s)$. This reduces the second ratio to 1 and leaves us with:

$$\mathrm{LR} = \frac{\Pr(Y = \Gamma_1|X = \Gamma_1, F^s)}{\Pr(Y = \Gamma_1|X = \Gamma_1, \bar{F}^s)} \tag{2}$$

The BN calculates these probabilities by instantiating the states figuring to the right of the vertical bar (i.e., setting their probabilities to 1). The BN then updates the probabilities of the states in noninstantiated nodes of the model according to the laws of probability. The numerator of the LR is, therefore, given by the probability of $Y = \Gamma_1$ after instantiating states $X = \Gamma_1$ and F^s (Fig. 2*a*). This probability is equal to 1 (as defined in Table 1, row 1, column 1). The denominator of the LR is given by the probability of $Y = \Gamma_1$ after instantiating states $X = \Gamma_1$ and $\bar{F}^s$ (Fig. 2*b*). This probability is equal to γ_i (as defined in Table 1, row 1, column 4). Thus the LR is

$$\mathrm{LR} = \frac{1}{\gamma_1}$$

which is the LR presented in forensic literature for a source level evaluation of a single trace (23).

Activity Level Bayesian Network for a Single Trace

Before addressing the two-trace transfer problem, this section presents the evaluation of an LR for activity level propositions in the case of a single trace. This explanation will be helpful in understanding the development of the LR for two traces. So, consider here the same scenario as described at the beginning of this study, but instead of recovering two traces on the crime scene, the investigators recover only a single trace. In this case, we consider the following pair of propositions (labeled *F*, because we are in a one-trace problem, with a superscript *a*, because they are activity level propositions):

F^a—*the suspect was engaged in a struggle with the victim at the location where the trace was recovered;*

$\bar{F}^a$—*the suspect was not engaged in a struggle with the victim at the location where the trace was recovered.*

To evaluate the LR for this pair of propositions, one must extend the BN in Fig. 1 to include transfer and background probabilities (12). For this, we must add a node *B* containing states:

B—presence of a background trace in the location where the trace was recovered;

$\bar{B}$—absence of a background trace in the location where the trace was recovered;

and a node *T* containing the states:

T—there was a transfer from the assailant during the struggle at the location where the trace was recovered;

$\bar{T}$—there was no transfer from the assailant during the struggle at the location where the trace was recovered.

Both of these will determine the characteristic of the trace we observe on the crime scene. If the trace is a background trace, the probability distribution over the states of *Y* will be equal to the match probabilities of the characteristic in the population of background traces. If the trace is a transferred trace, *Y* will have the characteristic of the assailant. Nodes *B* and *T* are therefore parents of node *Y* (Fig. 3*a*).

The characteristic of the assailant depends on whether the suspect is this assailant (node *F*). If F^a is true, the characteristic of the assailant is equal to the characteristic of the suspect's sample (node *X*), and if $\bar{F}^a$ is true, the probability distribution over the possible characteristics is given by the match probabilities in the population of possible assailants. To represent the characteristic of the assailant, we must create a new node containing the list of possible characteristics as its states, a node called *TS* for "true source" (12). This node is a child of *F* and *X* (Fig. 3*b*). Table 2 gives the conditional probability distribution over its states.

If the trace on the crime scene is a transferred trace, then its characteristic will be equal to the characteristic of the assailant

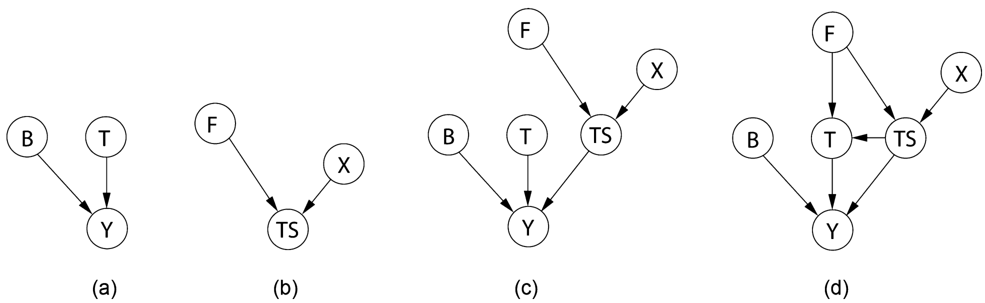

FIG. 3—*The construction of a BN for evaluating an activity level LR for a single trace (12). (a) The characteristic of the trace (node* Y*) depends on whether the trace is a background trace (node* B*) or a transferred trace (node* T*). (b) A transferred trace's true source (node* TS*) will only be equal to the suspect's characteristic (node* X*) if the suspect was the assailant (node* F*). (c) If the trace was a transferred trace, its characteristic will be equal to the characteristic in node* TS*. (d) The transfer probabilities (in node* T*) may differ according to the proposition in node* F *and the characteristic of the transferred trace's true source in node* TS*. This is the complete model for evaluating the activity level LR for a single trace (12). The nodes figuring in this BN are defined in Table 4.*

TABLE 2—*Probability table for node* TS *(true source) in Fig. 3. This node describes the characteristic of the transferred trace's true source. For simplicity, we use only three categories to describe the analytical results:* Γ_1, Γ_2, *and* Γ_{other} *(where* Γ_{other} *groups together all of the possible analytical results that are neither* Γ_1, *nor* Γ_2*).*

F:	F^a			$\bar{F}^a$		
X:	Γ_1	Γ_2	Γ_{other}	Γ_1	Γ_2	Γ_{other}
TS:						
Γ_1	1	0	0	γ_1	γ_1	γ_1
Γ_2	0	1	0	γ_2	γ_2	γ_2
Γ_{other}	0	0	1	$1-\gamma_1-\gamma_2$	$1-\gamma_1-\gamma_2$	$1-\gamma_1-\gamma_2$

given in node *TS*. Node *TS* is, therefore, a parent of node *Y* (Fig. 3*c*). Table 3 gives the conditional probability distribution over the states of *Y* given the states of its parents *T*, *B*, and *TS*.

The transfer probabilities defined in node *T* depend on the activity specified in node *F*. Sometimes, the activity described by proposition F^a will not be the same as the activity described by proposition $\bar{F}^a$. For example, the alternative proposition could describe a legitimate activity between the suspect and the victim (such as *the suspect was trying to rescue the victim*). In this case, the transfer probabilities will be different under each of the two propositions, making node *T* a child of node *F*. We therefore use t' (and its complement $\bar{t}'$) to denote the transfer probability given proposition $\bar{F}^a$ to distinguish this probability from *t* (and its complement $\bar{t}$) denoting the transfer probability given proposition F^a.

The occurrence of a transfer may also depend on the attributes of the transferred material. The extent of such an influence depends on the type of trace evidence considered. For example, for fiber evidence, a wool fiber may be transferred more easily than a silk fiber. If the analytical results in the BN are the types of fiber, then node *TS* must also be a parent of node *T* to specify different transfer probabilities for different types of fibers. Figure 3*d* shows the complete BN for the transfer of a single trace, modeling all of the possible dependence relationships (12).

If $X = \Gamma_1$ and $Y = \Gamma_1$, the LR computed by this BN is equal to

$$\mathrm{LR} = \frac{\Pr(Y=\Gamma_1|X=\Gamma_1,F^a)}{\Pr(Y=\Gamma_1|X=\Gamma_1,\bar{F}^a)} \tag{3}$$

$$= \frac{\bar{b}t + b\gamma_1'\bar{t}}{\bar{b}t'\gamma_1 + b\gamma_1'\bar{t}'} \tag{4}$$

This LR corresponds to the LR developed in the literature (5).

This BN computes the two probabilities of *Y* that form the LR in the same way as at the source level: the numerator is the probability of $Y = \Gamma_1$ when the states $X = \Gamma_1$ and F^a are instantiated and the denominator the probability of $Y = \Gamma_1$ when the states $X = \Gamma_1$ and $\bar{F}^a$ are instantiated (Fig. 4). However, unlike the source level evaluation, the resulting probabilities for $Y = \Gamma_1$ no longer figure in the probability table for *Y*. This is because in this BN, there are additional nodes separating node *Y* from nodes *X* and *F*. The calculation of the LR takes these intermediate nodes into account by extending the conversation (e.g., [25,26]) of the probability of *Y* to the parent variables of this node, that is, to *B*, *T* and *TS*, according to the relationships described in the probability table of node *Y* (Table 3). The first row of this table, corresponding to $Y = \Gamma_1$, tells us that the trace can only have characteristic Γ_1 when the trace was transferred from a source having characteristic Γ_1 in the absence of a background trace (column 4), or when the trace is a background trace and there was no transfer from the assailant (columns 7, 8, and 9). In all other cases, Table 3 defines a probability of 0 for observing $Y = \Gamma_1$, making this analytical result impossible for the combination of the states in that column. The table defines a probability of 1 for $Y = \Gamma_1$ if the trace was transferred, and a probability of γ_1' if the trace is a background trace, such that the LR (Eq. [3]) is equal to

$$\mathrm{LR} = \frac{1 \times \overbrace{\Pr(\bar{B}\cap T\cap TS=\Gamma_1|X=\Gamma_1,F^a)}^{(a)} + \gamma_i' \times \overbrace{\Pr(B\cap\bar{T}|X=\Gamma_1,F^a)}^{(b)}}{1 \times \underbrace{\Pr(\bar{B}\cap T\cap TS=\Gamma_1|X=\Gamma_1,\bar{F}^a)}_{(c)} + \gamma_i' \times \underbrace{\Pr(B\cap\bar{T}|X=\Gamma_1,\bar{F}^a)}_{(d)}} \tag{5}$$

where the mathematical symbol ∩ means the intersection where both the state on its left and the state on its right are true. The probabilities labeled (a), (b), (c), and (d) are discussed below:

(a) $\Pr(\bar{B}\cap T\cap TS=\Gamma_1|X=\Gamma_1,F^a)$ is the probability that the trace was transferred during the struggle from an assailant having characteristic Γ_1, given that the suspect has characteristic Γ_1 and that the suspect was this assailant who struggled with the victim. This probability is equal to

$$\bar{b}\times t\times\Pr(TS=\Gamma_1|X=\Gamma_1,F^a)$$

According to Table 2, $\Pr(TS = \Gamma_1|X = \Gamma_1, F^a) = 1$ (row 1, column 1), so that the above expression reduces to

$$\bar{b}\times t$$

(b) $\Pr(B\cap\bar{T}|X=\Gamma_1,F^a)$ is the probability that the trace is a background trace and that there was no transfer from the assailant, given that the suspect, having characteristic Γ_1, was the assailant in the struggle with the victim. As the trace was not transferred from the assailant, this probability is independent of the assailant's characteristic and is equal to

$$b\times\bar{t}$$

(c) $\Pr(\bar{B}\cap T\cap TS=\Gamma_1|X=\Gamma_1,\bar{F}^a)$ is the probability that the trace was transferred during the struggle from an assailant having characteristic Γ_1, given that the suspect has characteristic Γ_1, but that the suspect was not the assailant who struggled with the victim on the crime scene. This probability is equal to

$$\bar{b}\times t'\times\Pr(TS=\Gamma_1|X=\Gamma_1,\bar{F}^a)$$

According to Table 2, $\Pr(TS=\Gamma_1|X=\Gamma_1,\bar{F}^a)=\gamma_1$ (row 1, column 4), so that the above expression is equal to

$$\bar{b}\times t'\times\gamma_1$$

TABLE 3—*Probability table for node* Y *(the trace's characteristic) in Fig. 3. For simplicity, we use only three categories to describe the analytical results: Γ_1, Γ_2, and Γ_{other} (where Γ_{other} groups together all of the possible analytical results that are neither Γ_1, nor Γ_2). To complete this probability table, it is necessary to add the state "not 1 trace" to represent the events $T \cap B$ and $\bar{T} \cap \bar{B}$. However, as we observe* Y *as being only a single trace, this state does not enter into our calculations.*

T:	*T*						$\bar{T}$					
B:	*B*			$\bar{B}$			*B*			$\bar{B}$		
TS:	Γ_1	Γ_2	Γ_{other}	Γ_1	Γ_2	Γ_{other}	Γ_1	Γ_2	Γ_{other}	Γ_1	Γ_2	Γ_{other}
Y:												
Γ_1	0	0	0	1	0	0	γ'_1	γ'_1	γ'_1	0	0	0
Γ_2	0	0	0	0	1	0	γ'_2	γ'_2	γ'_2	0	0	0
Γ_{other}	0	0	0	0	0	1	$1-\gamma'_1-\gamma'_2$	$1-\gamma'_1-\gamma'_2$	$1-\gamma'_1-\gamma'_2$	0	0	0
not 1 trace	1	1	1	0	0	0	0	0	0	1	1	1

TABLE 4—*Description of the states and prior marginal probabilities of the nodes in Fig. 3. t and b denote the transfer and background probabilities, respectively. We differentiate between t, the transfer probability under F^a, and t′, the transfer probability under $\bar{F}^a$. For node* Y *(the trace's characteristic), we differentiate between γ_1 and γ_2, denoting the match probabilities in the population of potential assailants, used in the case the trace was transferred during the alleged activity, and γ'_1 and γ'_2, denoting the match probabilities in the population of background traces, used in the case that the trace is a background trace. For simplicity, we use only three categories to describe the analytical results: Γ_1, Γ_2, and Γ_{other} (where Γ_{other} groups together all of the possible analytical results that are neither Γ_1, nor Γ_2).*

Nodes	States	Prior Marginal Probabilities	Definitions of the States
F	F^a	$\Pr(F^a)$	The suspect was engaged in a struggle with the victim at the location where the trace was recovered
	$\bar{F}^a$	$1-\Pr(F^a)$	The suspect was not engaged in a struggle with the victim at the location where the trace was recovered
B	*B*	b	Presence of a background trace
	$\bar{B}$	$\bar{b}$	Absence of a background trace
T	*T*	t or t'	There was a transfer from the assailant
	$\bar{T}$	$\bar{t}$ or $\bar{t}'$	There was no transfer from the assailant
X	Γ_1	γ_1	Characteristic of the suspect's sample
	Γ_2	γ_2	
	Γ_{other}	$1-\gamma_1-\gamma_2$	
TS	Γ_1	γ_1	Characteristic of the trace's true source if it was transferred during the struggle with the victim
	Γ_2	γ_2	
	Γ_{other}	$1-\gamma_1-\gamma_2$	
Y	Γ_1	γ_1 or γ'_1	Characteristic of the trace
	Γ_2	γ_2 or γ'_1	
	Γ_{other}	$1-\gamma_1-\gamma_2$ or $1-\gamma'_1-\gamma'_2$	

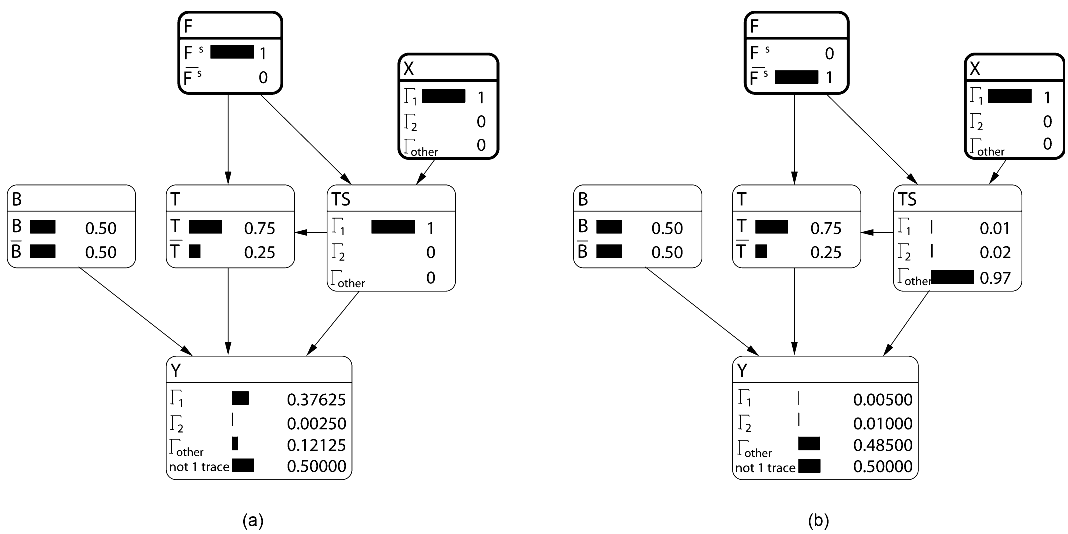

FIG. 4—*The BN in Fig. 3d computes the probabilities forming the LR* (Eq. [4]). *Here,* $b = 0.5$, $t = t' = 0.75$, $\gamma_1 = \gamma'_1 = 0.01$, *and* $\gamma_2 = \gamma'_2 = 0.02$. *The bold contour indicates that the node is instantiated. (a) The numerator of the LR is the probability of* $Y = \Gamma_1$ *when states* $X = \Gamma_1$ *and* F^a *are instantiated, in this case 0.37625; (b) the denominator is the probability of* $Y = \Gamma_1$ when states $X = \Gamma_1$ *and* $\bar{F}^a$ *are instantiated, in this case 0.005.*

(d) $\Pr(B \cap \bar{T} | X = \Gamma_1, \bar{F}^a)$ is the probability that the trace is a background trace and that there was no transfer from the assailant, given that the suspect, with characteristic Γ_1, was not the assailant in the struggle with the victim. As in (b), this probability is independent of the assailant's characteristic and is equal to

$$b \times \bar{t}\,'$$

Introducing the expressions for (a), (b), (c), and (d) into Eq. (5) produces

$$\mathrm{LR} = \frac{1 \times \bar{b} \times t + \gamma_1' \times b \times \bar{t}}{1 \times \bar{b} \times t' \times \gamma_1 + \gamma_1' \times b \times \bar{t}\,'}$$

which is Eq. (4). This calculation validates the structure of the BN in Fig. 3*d* for evaluating an activity level LR for a single trace. In the next section, we extend this BN to two traces.

Constructing a Bayesian Network for Two Traces

This section describes the steps for extending a BN for a single trace to two traces. We first illustrate this concept for the BN in Fig. 1, at the source level, and then apply the same reasoning to the BN in Fig. 3*d*, at the activity level, to address the two-trace transfer problem described at the beginning of the paper.

Constructing a Source Level Bayesian Network for Two Traces

In a two-trace problem, the evidence consists of the characteristic of the suspect's sample and the characteristics of two traces recovered on the crime scene. We will call the first trace recovered on the crime scene "trace 1," and the second trace recovered on the scene "trace 2."

To represent the characteristics of each of the traces, we duplicate node Y in Fig. 1, creating a node for trace 1 called Y_1, and a node for trace 2 called Y_2. As in Fig. 1, the characteristic of each of these traces will be identical to the characteristic of the suspect's sample if that trace comes from the suspect. Therefore, Y_1 and Y_2 are each a child of nodes X and F. To distinguish between the two traces, we duplicate node F to create node F_1 for trace 1, and node F_2 for trace 2 (Fig. 5*a*), so that F_1 contains the propositions:

F_1^s—*trace 1 comes from the suspect;*
$\bar{F}_1^s$—*trace 1 does not come from the suspect*;

and F_2 the propositions:

F_2^s—*trace 2 comes from the suspect;*
$\bar{F}_2^s$—*trace 2 does not come from the suspect.*

The probability tables of nodes Y_1 and Y_2 are identical to Table 1 for node Y in a one-trace scenario.

Now, the propositions of interest for evaluating an LR for multiple traces are no longer F^s and $\bar{F}^s$, but H^s and $\bar{H}^s$:

H^s—*one of the traces on the crime scene comes from the suspect;*
$\bar{H}^s$—*neither of the traces on the crime scene comes from the suspect.*

Assuming that it is not possible for both of the traces on the crime scene to come from the suspect, and that it is equally probable for either of the traces to come from the suspect when H^s is true (for a source level BN where we relax these assumptions, see [27]), then either F_1^s or F_2^s must be true when proposition H^s is true, but F_1^s and F_2^s will never both be true at the same time. We therefore add node H, containing states H^s and $\bar{H}^s$, as a parent of nodes F_1 and F_2. In addition, we must add a link between nodes

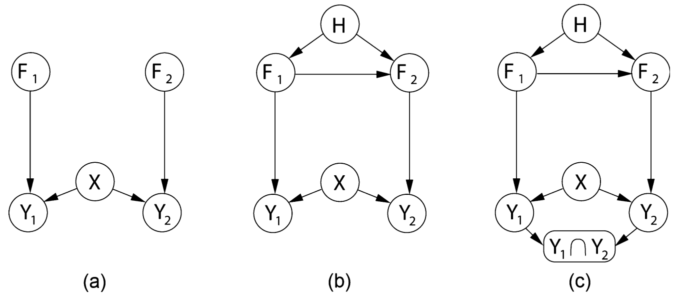

FIG. 5—*The construction of a BN for evaluating a source level LR for two traces. (a) Nodes* F *and* Y *in Fig. 1 are duplicated such that nodes* F_1 *and* Y_1 *refer to trace 1, and nodes* F_2 *and* Y_2 *to trace 2. (b) The pair of propositions* H^s *and* $\bar{H}^s$ *in node* H *are added as a parent to nodes* F_1 *and* F_2 *(see Tables 5 and 6 for the conditional probability tables of* F_1 *and* F_2*). (c) The BN is completed by adding a node* $Y_1 \cap Y_2$ *to compute the numerator and denominator of the LR. This is a modified version of the BN presented in (12).*

F_1 and F_2 to assure that F_2^s is true (i.e., trace 2 comes from the suspect) when H^s (i.e., one of the traces comes from the suspect) and $\bar{F}_1^s$ (i.e., trace 1 does not come from the suspect) are both true (Fig. 5*b*). If proposition $\bar{H}^s$ is true, then it follows that both $\bar{F}_1^s$ and $\bar{F}_2^s$ are true. Tables 5 and 6 define the probability distributions over F_1^s, $\bar{F}_1^s$, F_2^s, and $\bar{F}_2^s$.

For this BN to compute the numerator and denominator of the LR, we add a node that combines the characteristics of both traces, node $Y_1 \cap Y_2$, as a child of Y_1 and Y_2 (Fig. 5*c*). The resulting BN is a slightly modified version of the BN presented in (12). (In [12], propositions F_1^s, $\bar{F}_1^s$, F_2^s, and $\bar{F}_2^s$ are all combined in a single node F.) The two versions are logically equivalent and compute an LR of

$$\mathrm{LR} = \frac{\Pr(Y_1 = \Gamma_1, Y_2 = \Gamma_2 | X = \Gamma_1, H^s)}{\Pr(Y_1 = \Gamma_1, Y_2 = \Gamma_2 | X = \Gamma_1, \bar{H}^s)} = \frac{1}{2\gamma_1}$$

for $Y_1 = \Gamma_1$, $Y_2 = \Gamma_2$, and $X = \Gamma_1$. This model is, therefore, in perfect agreement with scientific literature (3). For further explanations or examples of a BN treating a two-trace problem at the source level, see (12,27).

TABLE 5—*Probability table for node* F_1 *in Fig. 5. This node indicates whether or not trace 1 comes from the suspect. Given that one of the traces on the crime scene comes from the suspect, this probability table considers it equally probable for this trace to be trace 1 or trace 2.*

H:	H^s	$\bar{H}^s$
F_1:		
F_1^s	0.5	0
$\bar{F}_1^s$	0.5	1

TABLE 6—*Probability table for node* F_2 *in Fig. 5. This node indicates whether or not trace 2 comes from the suspect. This probability table considers it impossible for both of the traces to come from the suspect, and considers that either trace 1 or trace 2 must come from the suspect if* H^s *is true.*

H:	H^s		$\bar{H}^s$	
F_1:	F_1^s	$\bar{F}_1^s$	F_1^s	$\bar{F}_1^s$
F_2:				
F_2^s	0	1	0	0
$\bar{F}_2^s$	1	0	1	1

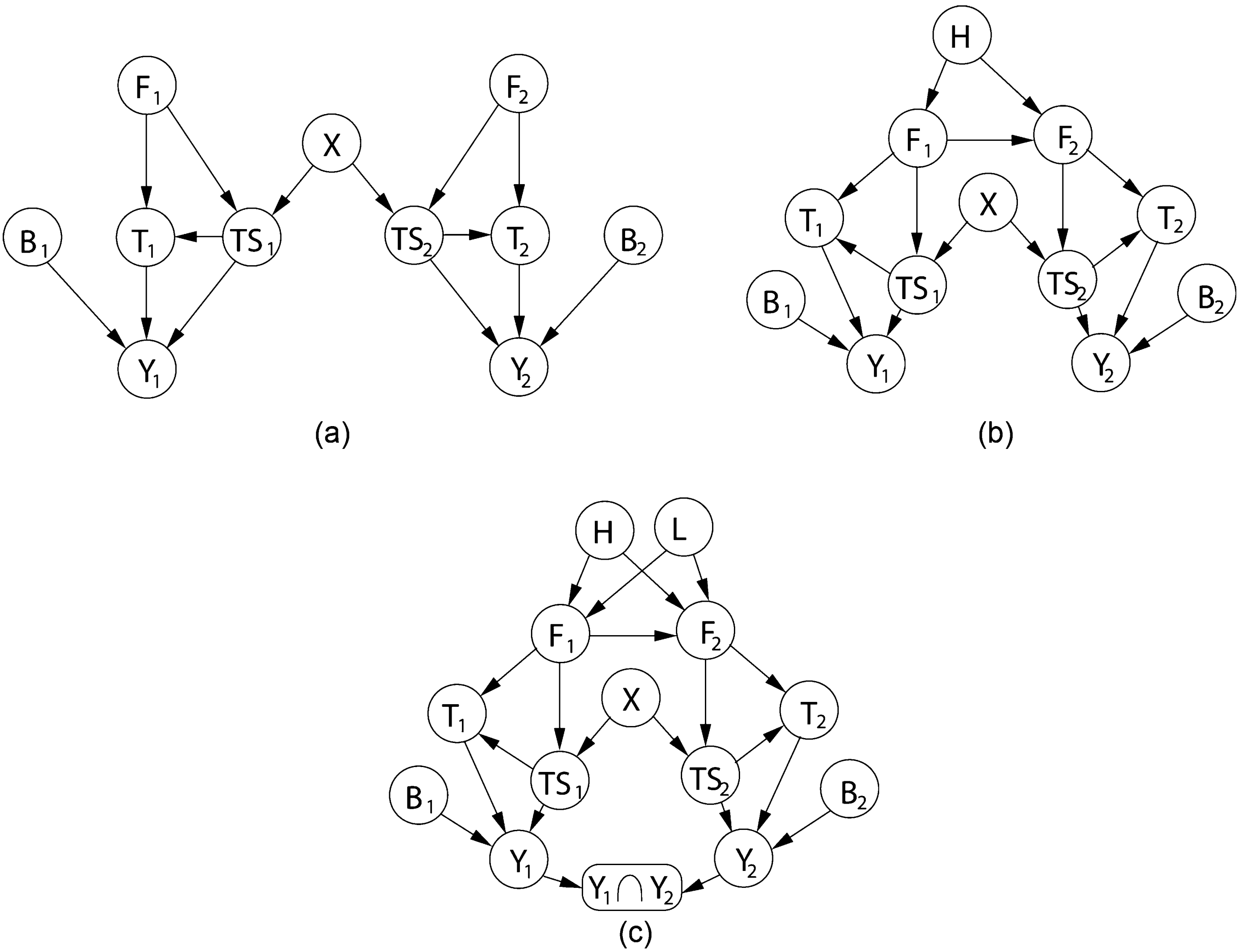

FIG. 6—*The construction of a BN for evaluating an activity level LR for two traces. We begin with the BN in Fig. 3d. (a) We duplicate all of the nodes in this model except for* X, *and use subscripts 1 and 2 to differentiate the nodes referring to trace 1 from the nodes referring to trace 2. (b) We add a node* H and an arrow from F_1 to F_2 *just as in Fig. 5b. (c) We add an additional node* L *to model the uncertainty on whether the assailant in location 1 was the same person as the assailant in location 2, and a node* $Y_1 \cap Y_2$ *to compute the numerator and denominator of the LR.*

Constructing an Activity Level Bayesian Network for Two Traces

To address the two-trace transfer problem at the activity level, we proceed in the same way as at the source level to extend a BN of a single trace to two traces. We begin by using node X as the center of the new model, and duplicate the rest of the nodes on either side of X, such that we have a set of nodes referring to trace 1 (labeled with a subscript 1) on the left of X, and a set of nodes referring to trace 2 (labeled with a subscript 2) on the right of X (Fig. 6*a*). The probability tables for nodes TS_1, TS_2, Y_1, and Y_2 are identical to Tables 2 and 3, for nodes TS and Y, respectively.

Again, we introduce a node H as a parent of nodes F_1 and F_2. H contains the propositions of interest for the two-trace transfer problem:

H^a—*the suspect was engaged in a struggle with the victim;*
$\bar{H}^a$—*the suspect was not engaged in a struggle with the victim;*

F_1 the propositions:

F_1^a—*the suspect was engaged in a struggle with the victim in location 1;*
$\bar{F}_1^a$—*the suspect was not engaged in a struggle with the victim in location 1;*

and F_2 the propositions:

$\bar{F}_2^a$—*the suspect was engaged in a struggle with the victim in location 2;*
$\bar{F}_2^a$—*the suspect was not engaged in a struggle with the victim in location 2.*

If $\bar{H}^a$ is true, then it follows that both $\bar{F}_1^a$ and $\bar{F}_2^a$ are true. If H^a is true, we assume that either F_1^a, F_2^a, or both F_1^a and F_2^a must be true. Unlike the BN we presented above for a source level evaluation, we will take into account here the possibility that it may have been the same assailant in both locations. For this, we must link F_1 to F_2, as before (Fig. 6*b*), and define a new node L containing the states

L—*the assailant in location 1 is the same person as the assailant in location 2;*

TABLE 7—*Probability table for node* F_1 *in Fig. 6. This node indicates whether or not the suspect was engaged in a struggle with the victim in location 1. We use* τ *to denote the probability that the suspect was engaged in the struggle in location 1 given that he is one of two assailants (i.e.,* $\Pr(F_1^a|\bar{L}, H^a)$*).*

H:	H^a		$\bar{H}^a$	
L:	L	$\bar{L}$	L	$\bar{L}$
F_1:				
F_1^a	1	τ	0	0
$\bar{F}_1^a$	0	$1-\tau$	1	1

TABLE 8—*Probability table for node* F_2 *in Fig. 6. This node indicates whether or not the suspect was engaged in a struggle with the victim in location 2. This is only possible when either the suspect was the assailant in both locations (column 1), or when the suspect was one of two assailants and he was not engaged in a struggle with the victim in location 1 (column 4).*

H:	H^a				$\bar{H}^a$			
L:	L		$\bar{L}$		L		$\bar{L}$	
F_1:	F_1^a	$\bar{F}_1^a$	F_1^a	$\bar{F}_1^a$	F_1^a	$\bar{F}_1^a$	F_1^a	$\bar{F}_1^a$
F_2:								
F_2^a	1	n/a	0	1	0	0	0	0
$\bar{F}_2^a$	0	n/a	1	0	1	1	1	1

$\bar{L}$—the assailant in location 1 is not the same person as the assailant in location 2;

as a parent to F_1 and F_2 (27). We use the probability λ to denote the prior probability that there were two different assailants on the crime scene (28), that is, $\Pr(\bar{L}) = \lambda$. To relax the assumption that it is equally probable for the suspect to have struggled with the victim in either of the two locations in a case where the suspect was one of two assailants, we introduce another probability, which we call τ. This probability represents the prior probability that the suspect was engaged in the struggle in location 1 in the case that the suspect is an assailant and that there were two different assailants in locations 1 and 2 on the crime scene, i.e., $\Pr(F_1^a|\bar{L}, H^a) = \tau$. Tables 7 and 8 give the probability tables for F_1 and F_2.

To compute the LR,

$$\mathrm{LR} = \frac{\Pr(Y_1 = \Gamma_1, Y_2 = \Gamma_2 | X = \Gamma_1, H^a)}{\Pr(Y_1 = \Gamma_1, Y_2 = \Gamma_2 | X = \Gamma_1, \bar{H}^a)} \tag{6}$$

we add a node $Y_1 \cap Y_2$ as a child of nodes Y_1 and Y_2 (Fig. 6*c*). This completes the BN, with all of the nodes and states of the final model given in Table 9.

Note that if the same background noise is present in both locations, nodes B_1 and B_2 may be merged into a single node B, parent to both Y_1 and Y_2. This creates an additional probabilistic link between Y_1 and Y_2, which influences the evaluation of the LR. However, this special case is not treated in this paper.

In the next section, we present the algebraic expression that corresponds to the computed LR. We derive this expression to compare the LR provided by this model with the existing formulae in forensic literature.

Algebraic Expression for the LR Computed by the Bayesian Network in Fig. 6*c*

The robust mathematical framework of BNs allows their user to deduce the algebraic formulae of interest. In the forensic context of the two-trace transfer problem, we are interested in the LR for propositions H^a and $\bar{H}^a$ (Eq. [6]). In this section, we introduce the transfer, background, and match probabilities into this expression, based on the probabilistic relationships modeled in the BN in Fig. 6*c*. For this development, we assume that trace 1 has characteristic Γ_1 ($Y_1 = \Gamma_1$) and trace 2 characteristic Γ_2 ($Y_2 = \Gamma_2$). As for the characteristic of the suspect's sample, we will label it X for the time being and specify its characteristic later on.

We begin by applying the third law of probability for dependent events to Eq. (6):

$$\mathrm{LR} = \underbrace{\frac{\Pr(Y_1 = \Gamma_1 | X, H^a)}{\Pr(Y_1 = \Gamma_1 | X, \bar{H}^a)}}_{\text{(i)}} \times \underbrace{\frac{\Pr(Y_2 = \Gamma_2 | Y_1 = \Gamma_1, X, H^a)}{\Pr(Y_2 = \Gamma_2 | Y_1 = \Gamma_1, X, \bar{H}_a)}}_{\text{(ii)}} \tag{7}$$

This separates the LR into the product of two ratios:

(i) the LR for observing the characteristic of trace 1 for the two competing propositions given the characteristic of the suspect's sample;

(ii) the LR for observing the characteristic of trace 2 for the same propositions, given the characteristic of the suspect's sample *and given that the characteristic of trace 1 has already been observed.*

As the probability tables for nodes Y_1 and Y_2 are identical to the probability table for node Y in the one-trace evaluation, the developments of these two ratios over nodes B_1, T_1 and TS_1, and B_2, T_2 and TS_2, respectively, are identical to Eq. (5):

$$\mathrm{LR} = \frac{1 \times \overbrace{\Pr(\bar{B}_1 \cap T_1 \cap TS_1 = \Gamma_1 | X, H^a)}^{\text{(a)}} + \gamma_1' \times \overbrace{\Pr(B_1 \cap \bar{T}_1 | X, H^a)}^{\text{(b)}}}{1 \times \underbrace{\Pr(\bar{B}_1 \cap T_1 \cap TS_1 = \Gamma_1 | X, \bar{H}^a)}_{\text{(c)}} + \gamma_1' \times \underbrace{\Pr(B_1 \cap \bar{T}_1 | X, \bar{H}^a)}_{\text{(d)}}}$$

$$\times \frac{1 \times \overbrace{\Pr(\bar{B}_2 \cap T_2 \cap TS_2 = \Gamma_2 | Y_1 = \Gamma_1, X, H^a)}^{\text{(e)}} + \gamma_2' \times \overbrace{\Pr(B_2 \cap \bar{T}_2 | Y_1 = \Gamma_1, X, H^a)}^{\text{(f)}}}{1 \times \underbrace{\Pr(\bar{B}_2 \cap T_2 \cap TS_2 = \Gamma_2 | Y_1 = \Gamma_1, X, \bar{H}^a)}_{\text{(g)}} + \gamma_2' \times \underbrace{\Pr(B_2 \cap \bar{T}_2 | Y_1 = \Gamma_1, X, \bar{H}^a)}_{\text{(h)}}} \tag{8}$$

Let us first examine the denominators of these two ratios.

Denominators of the Ratios in Eq. (8)

Under $\bar{H}^a$, neither of the traces was transferred by the suspect, so we assume the suspect's characteristic has no influence on the probabilities of $Y_1 = \Gamma_1$ and $Y_2 = \Gamma_2$, and remove the conditioning on X from these probabilities.

Probability (c) (i.e., $\Pr(\bar{B}_1 \cap T_1 \cap TS_1 = \Gamma_1 | \bar{H}^a)$) is the probability that trace 1 was transferred during the struggle from an assailant having characteristic Γ_1, given that the suspect was not an assailant who struggled with the victim on the crime scene. As in the one-trace transfer problem, this probability is equal to

TABLE 9—*Description of the states and prior marginal probabilities of the nodes in Fig. 6, with i = 1, 2. We use* δ *to denote the prior probability that the suspect was engaged in a struggle with the victim (i.e.,* $Pr(H^a) = \delta$*),* λ *to denote the prior probability that the victim struggled with two different assailants in location 1 and location 2 (i.e.,* $\Pr(\bar{L}) = \lambda$*), and* τ *to denote the prior probability of the suspect being the assailant who struggled with the victim in location 1 in the case where the suspect was an assailant, and the assailant in location 1 was not the same person as the assailant in location 2 (i.e.,* $\Pr(F_1^a|\bar{L}, H^a) = \tau$*). For node* T_i*, we differentiate between* t_i*, the transfer probability under* F_i^a*, and* t_i'*, the transfer probability under* $\bar{F}_i^a$*. For node Y (the trace's characteristic), we differentiate between* γ_1 *and* γ_2*, denoting the match probabilities in the population of potential assailants, used in the case the trace was transferred during the alleged activity, and* γ_1' *and* γ_2'*, denoting the match probabilities in the population of background traces, used in the case that the trace is a background trace. For simplicity, we use only three categories to describe the analytical results:* Γ_1*,* Γ_2*, and* Γ_{other} *(where* Γ_{other} *groups together all of the possible analytical results that are neither* Γ_1*, nor* Γ_2*).*

Nodes	States	Prior Marginal Probabilities	Definitions of the States
H	H^a	δ	The suspect was engaged in a struggle with the victim
	$\bar{H}^a$	$1 - \delta$	The suspect was not engaged in a struggle with the victim
L	L	$1 - \lambda$	The assailant in location 1 is the same person as the assailant in location 2
	$\bar{L}$	λ	The assailant in location 1 is not the same person as the assailant in location 2
F_1	F_1^a	$\delta\ (1 - \lambda + \lambda\tau)$	The suspect was engaged in a struggle with the victim in location 1
	$\bar{F}_1^a$	$\delta\lambda\ (1 - \tau) + 1 - \delta$	The suspect was not engaged in a struggle with the victim in location 1
F_2	F_2^a	$\delta\ (1 - \lambda + \lambda(1 - \tau))$	The suspect was engaged in a struggle with the victim in location 2
	$\bar{F}_2^a$	$\delta\lambda\tau + 1 - \delta$	The suspect was not engaged in a struggle with the victim in location 2
B_i	B_i	b_i	Presence of a background trace in location i
	$\bar{B}_i$	$\bar{b}_i$	Absence of a background trace in location i
T_i	T_i	t_i or t_i'	There was a transfer from the assailant in location i
	$\bar{T}_i$	$\bar{t}_i$ or $\bar{t}_i'$	There was no transfer from the assailant in location i
X	Γ_1	γ_1	Characteristic of the suspect's sample
	Γ_2	γ_2	
	Γ_{other}	$1 - \gamma_1 - \gamma_2$	
TS_i	Γ_1	γ_1	Characteristic of trace i's true source if it was transferred during the struggle with the victim
	Γ_2	γ_2	
	Γ_{other}	$1 - \gamma_1 - \gamma_2$	
Y_i	Γ_1	γ_1 or γ_1'	Characteristic of trace i
	Γ_2	γ_2 or γ_2'	
	Γ_{other}	$1 - \gamma_1 - \gamma_2$ or $1 - \gamma_1' - \gamma_2'$	
$Y_1 \cap Y_2$	Combine Y_1 and Y_2		Characteristics of trace 1 and trace 2

$$\bar{b}_1 \times t_1' \times \Pr(TS_1 = \Gamma_1|\bar{H}^a)$$

The probability table for node TS_1 is identical to the probability table for node TS shown in Table 2, such that $\Pr(TS_1 = \Gamma_1|\bar{H}^a) = \gamma_1$, and the above expression is equal to

$$\bar{b}_1 \times t_1' \times \gamma_1$$

Probability (d) (i.e., $\Pr(B_1 \cap \bar{T}_1|\bar{H}^a)$) is the probability that trace 1 is a background trace and that there was no transfer from the assailant in location 1, given that the suspect was not an assailant in the struggle with the victim. As in the one-trace transfer problem, this probability is independent of the assailant's characteristic and is equal to

$$b_1 \times t_1'$$

Probability (g) (i.e., $\Pr(\bar{B}_2 \cap T_2 \cap TS_2 = \Gamma_2|Y_1 = \Gamma_1, \bar{H}^a)$) is the probability that trace 2 was transferred during the struggle from an assailant having characteristic Γ_2, given that trace 1 has characteristic Γ_1 and that the suspect was not an assailant who struggled with the victim on the crime scene. This probability is equal to

$$\bar{b}_2 \times t_2' \times \Pr(TS_2 = \Gamma_2|Y_1 = \Gamma_1, \bar{H}^a)$$

The probability table for node TS_2 is identical to the probability table for node TS shown in Table 2, such that $\Pr(TS_2 = \Gamma_2|Y_1 = \Gamma_1, \bar{H}^a) = \gamma_2$, and the above expression is equal to

$$\bar{b}_2 \times t_2' \times \gamma_2$$

Probability (h) (i.e., $\Pr(B_2 \cap \bar{T}_2|Y_1 = \Gamma_1, \bar{H}^a)$) is the probability that trace 2 is a background trace and that there was no transfer from the assailant in location 2, given that trace 1 has characteristic Γ_1 and that the suspect was not an assailant in the struggle with the victim. As in the development of probability (d), this probability is independent of the assailant's characteristic and is equal to

$$b_2 \times \bar{t}_2'$$

The denominator of ratio (i) is therefore

$$\bar{b}_1 t_1' \gamma_1 + \gamma_1' b_1 \bar{t}_1'$$

and the denominator for ratio (ii),

$$\bar{b}_2 t_2' \gamma_2 + \gamma_2' b_2 \bar{t}_2'.$$

Each of these expressions is identical to the denominator of the LR published for the one-trace transfer problem (Eq. [4]). This is reasonable, because the observation of two different traces makes it impossible for the two traces to have come from the same person, such that the observations of the two traces can be considered independent of each other. Each observation is, therefore, comparable with the observation of a single trace.

Next, let us examine the numerators of the two ratios in Eq. (8).

Numerators of the Ratios in Eq. (8)

Probabilities (a) and (e) (i.e., $\Pr(\bar{B}_1 \cap T_1 \cap TS_1 = \Gamma_1|X, H^a)$ and $\Pr(\bar{B}_2 \cap T_2 \cap TS_2 = \Gamma_2|Y_1 = \Gamma_1, X, H^a)$) are the probabilities that each of the traces was transferred during the struggle, and probabilities (b) and (f) (i.e., $\Pr(B_1 \cap \bar{T}_1|X, H^a)$ and $\Pr(B_2 \cap \bar{T}_2|Y_1 = \Gamma_1, X, H^a)$) the probabilities that each of the traces is a background trace and that there was no transfer from the struggle in each of the locations. The developments of the latter are independent of an assailant's characteristic. As in the one-trace transfer problem, probability (b) is equal to

$$b_1 \times \bar{t}_1$$

and probability (f) is equal to

$$b_2 \times \bar{t}_2$$

Concerning the probabilities that each of the traces was transferred during the struggle under H^a, it is possible that a trace may have been transferred by the suspect. We must, therefore, take into account the probability that the true source of a transferred trace may effectively be the suspect. As a result, we must consider the characteristic of the suspect's sample to develop probabilities (a) and (e).

Theoretically, there are two possibilities for the evidence in a two-trace problem:

(1) the suspect's sample has characteristic Γ_1 (i.e., $X = \Gamma_1$) and matches trace 1; or
(2) the suspect's sample has characteristic Γ_2 (i.e., $X = \Gamma_2$) and matches trace 2.

In the first case, the suspect matches the first trace observed on the scene; in the second case, the suspect matches the second trace observed on the scene. Of course, the order of the observations will not affect the numerical value obtained for the LR. However, the algebraic derivation of the formulae includes conditional probabilities which will differ in these two scenarios. Therefore, we will develop probabilities (a) and (e) twice: first, we will assume that the suspect's sample matches the first trace, and second, that the suspect's sample matches the second trace.

In the Case that the Suspect's Sample Matches Trace 1 (i.e., $X = \Gamma_1$)

Probability (a), that is, $\Pr(\bar{B}_1 \cap T_1 \cap TS_1 = \Gamma_1 | X = \Gamma_1, H^a)$, is the probability of the event that trace 1 was transferred during the struggle from an assailant having characteristic Γ_1, given that the suspect has characteristic Γ_1 and that the suspect was an assailant who struggled with the victim on the crime scene. As in the one-trace transfer problem, this probability is equal to

$$\bar{b}_1 \times t_1 \times \Pr(TS_1 = \Gamma_1 | X = \Gamma_1, H^a) \tag{9}$$

To find $\Pr(TS_1 = \Gamma_1 | X = \Gamma_1, H^a)$, we must continue to work our way up in the structure of the BN (Fig. 6*c*). Between nodes TS_1 and H is node F_1. We must, therefore, extend the conversation to propositions F_1^a and $\bar{F}_1^a$:

$$\begin{aligned}\Pr(TS_1 = \Gamma_1 | X = \Gamma_1, H^a) &= 1 \times \underbrace{\Pr(F_1^a | H^a)}_{1-\lambda+\lambda\tau} + \gamma_1 \times \underbrace{\Pr(\bar{F}_1^a | H^a)}_{\lambda(1-\tau)} \\ &= 1 - \lambda + \lambda\tau + \gamma_1 \lambda (1 - \tau)\end{aligned}$$

This development results from two possible explanations for the event $TS_1 = \Gamma_1$ (i.e., the event that a transferred trace's true source in location 1 is Γ_1):

- either trace 1 was transferred from the suspect, in which case the probability of $TS_1 = \Gamma_1$ is equal to 1;
- or trace 1 was transferred by the other assailant, who has characteristic Γ_1 with a probability of γ_1.

The weighted sum of these values with the probabilities of each of them occurring given proposition H^a produces the above expression. The first case is possible either when the suspect was the only assailant (probability of $1 - \lambda$), or when the suspect was one of two assailants (probability of λ) and of these two, was the one in location 1 (probability of τ). The second case is only possible when the suspect was one of two assailants (probability of λ), and this time was the one in location 2 (probability of $1 - \tau$).

Inserting these results into Eq. (9) gives us the following expression for $\Pr(\bar{B}_1 \cap T_1 \cap TS_1 = \Gamma_1 | X = \Gamma_1, H^a)$:

$$\bar{b}_1 \times t_1 \times [1 - \lambda + \lambda\tau + \gamma_1 \lambda (1 - \tau)]$$

Introducing the expressions for (a), (b), (c) and (d) into Eq. (8) produces

$$\frac{\Pr(Y_1 = \Gamma_1 | X = \Gamma_1, H^a)}{\Pr(Y_1 = \Gamma_1 | X = \Gamma_1, \bar{H}^a)} = \frac{\bar{b}_1 t_1 [1 - \lambda + \lambda\tau + \gamma_1 \lambda (1 - \tau)] + \gamma_1' b_1 \bar{t}_1}{\bar{b}_1 t_1' \gamma_1 + \gamma_1' b_1 \bar{t}_1'} \tag{10}$$

for ratio (i) in Eq. (7).

The expression for (e) is more complex. $\Pr(\bar{B}_2 \cap T_2 \cap TS_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_1, H^a)$ is the probability of the event that trace 2 was transferred during the struggle from an assailant having characteristic Γ_2, given that trace 1 and the suspect have characteristic Γ_1 and that the suspect was an assailant who struggled with the victim on the crime scene. This probability is equal to

$$\bar{b}_2 \times t_2 \times \Pr(TS_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_1, H^a) \tag{11}$$

and the extension of the conversation to propositions F_2^a and $\bar{F}_2^a$ to find $\Pr(TS_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_1, H^a)$ produces

$$\begin{aligned}&\Pr(TS_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_1, H^a) \\ &\quad = \gamma_2 \times \Pr(\bar{F}_2^a | Y_1 = \Gamma_1, X = \Gamma_1, H^a)\end{aligned} \tag{12}$$

Given that the suspect has characteristic Γ_1, the true source of a transferred trace in location 2 can only have characteristic Γ_2 if the assailant in this location was not the suspect (proposition $\bar{F}_2^a$). In this case, the probability that the true source has characteristic Γ_2 is γ_2. To compute $\Pr(\bar{F}_2^a | Y_1 = \Gamma_1, X = \Gamma_1, H^a)$, the BN applies Bayes' theorem:

$$\begin{aligned}&\Pr(\bar{F}_2^a | Y_1 = \Gamma_1, X = \Gamma_1, H^a) \\ &\quad = \frac{\Pr(Y_1 = \Gamma_1 | \bar{F}_2^a, X = \Gamma_1, H^a) \times \Pr(\bar{F}_2^a | H^a)}{\Pr(Y_1 = \Gamma_1 | X = \Gamma_1, H^a)}\end{aligned} \tag{13}$$

This ratio is made up of the following three probabilities:

- The development of probability $\Pr(Y_1 = \Gamma_1 | \bar{F}_2^a, X = \Gamma_1, H^a)$ is identical to that of the numerator of the traditional one-trace transfer scenario (Eq. [5]):

$$\begin{aligned}&\Pr(Y_1 = \Gamma_1 | \bar{F}_2^a, X = \Gamma_1, H^a) \\ &\quad = 1 \times \Pr(\bar{B}_1 \cap T_1 \cap TS_1 = \Gamma_1 | \bar{F}_2^a, X = \Gamma_1, H^a) \\ &\quad + \gamma_1' \times \Pr(B_1 \cap \bar{T}_1 | H^a) \\ &\quad = \bar{b}_1 t_1 + \gamma_1' b_1 \bar{t}_1\end{aligned}$$

- By definition,

$$\Pr(\bar{F}_2^a | H^a) = \lambda\tau$$

that is, the probability that the assailant in location 2 was not the suspect given that the suspect was an assailant is equal to the probability that there were two different assailants in each of the locations (probability λ) and that the suspect was the assailant in location 1 (probability τ).

- And Pr ($Y_1 = \Gamma_1 | X = \Gamma_1, H^a$) is the numerator of the first ratio in our LR (Eq. [10]);

$$\Pr(Y_1 = \Gamma_1 | X = \Gamma_1, H^a) = \bar{b}_1 t_1 [1 - \lambda + \lambda\tau + \gamma_1 \lambda(1-\tau)] + \gamma'_1 b_1 \bar{t}_1$$

Therefore, Eq. (13) is equal to

$$\Pr(\bar{F}^a_2 | Y_1 = \Gamma_1, X = \Gamma_1, H^a) = \frac{(\bar{b}_1 t_1 + \gamma'_1 b_1 \bar{t}_1)\lambda\tau}{\bar{b}_1 t_1 [1 - \lambda + \lambda\tau + \gamma_1 \lambda(1-\tau)] + \gamma'_1 b_1 \bar{t}_1}$$

Inserting this result into Eq. (12), and Eq. (12) into Eq. (11), makes probability (e) in Eq. (8) equal to

$$\begin{aligned}&\Pr(\bar{B}_2 \cap T_2 \cap TS_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_1, H^a) \\ &\quad = \bar{b}_2 t_2 \gamma_2 \left\{ \frac{(\bar{b}_1 t_1 + \gamma'_1 b_1 \bar{t}_1)\lambda\tau}{\bar{b}_1 t_1 [1 - \lambda + \lambda\tau + \gamma_1 \lambda(1-\tau)] + \gamma'_1 b_1 \bar{t}_1} \right\}\end{aligned}$$

Introducing the expressions for (e), (f), (g), and (h) into Eq. (8) produces

$$\frac{\Pr(Y_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_1, H^a)}{\Pr(Y_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_1, \bar{H}^a)} = \frac{\bar{b}_2 t_2 \gamma_2 \left\{ \frac{(\bar{b}_1 t_1 + \gamma'_1 b_1 \bar{t}_1)\lambda\tau}{\bar{b}_1 t_1 [1 - \lambda + \lambda\tau + \gamma_1 \lambda(1-\tau)] + \gamma'_1 b_1 \bar{t}_1} \right\} + \gamma'_2 b_2 \bar{t}_2}{\bar{b}_2 t'_2 \gamma_2 + \gamma'_2 b_2 \bar{t}'_2}$$

for ratio (ii) in Eq. (7).

Inserting all of the obtained results into Eq. (8) leads to the following activity level LR for a case in which the suspect's sample matches the first of two traces recovered on a crime scene:

$$\mathrm{LR} = \underbrace{\frac{\bar{b}_1 t_1 [1 - \lambda + \lambda\tau + \gamma_1 \lambda(1-\tau)] + \gamma'_1 b_1 \bar{t}_1}{\bar{b}_1 t'_1 \gamma_1 + \gamma'_1 b_1 \bar{t}'_1}}_{(i)} \times \underbrace{\frac{\bar{b}_2 t_2 \gamma_2 \left\{ \frac{(\bar{b}_1 t_1 + \gamma'_1 b_1 \bar{t}_1)\lambda\tau}{\bar{b}_1 t_1 [1 - \lambda + \lambda\tau + \gamma_1 \lambda(1-\tau)] + \gamma'_1 b_1 \bar{t}_1} \right\} + \gamma'_2 b_2 \bar{t}_2}{\bar{b}_2 t'_2 \gamma_2 + \gamma'_2 b_2 \bar{t}'_2}}_{(ii)}$$

In the Case that the Suspect's Sample Matches Trace 2 (i.e., $X = \Gamma_2$)

In this case, probability (a), that is, $\Pr(\bar{B}_1 \cap T_1 \cap TS_1 = \Gamma_1 | X = \Gamma_2, H^a)$, is the probability of the event that trace 1 was transferred during the struggle from an assailant having characteristic Γ_1, given that the suspect was an assailant who struggled with the victim on the crime scene, but that the suspect has characteristic Γ_2. This probability is equal to

$$\bar{b}_1 \times t_1 \times \Pr(TS_1 = \Gamma_1 | X = \Gamma_2, H^a)$$

In this case, the true source can only have characteristic Γ_1 if the suspect was not the assailant in location 1 (proposition $\bar{F}^a_1$), and if the assailant in location 1 possesses characteristic Γ_1 (probability of γ_1):

$$\begin{aligned}\Pr(TS_1 = \Gamma_1 | X = \Gamma_1, H^a) &= \gamma_1 \times \underbrace{\Pr(\bar{F}^a_1 | H^a)}_{\lambda(1-\tau)} \\ &= \gamma_1 \lambda(1-\tau)\end{aligned}$$

The expression for $\Pr(\bar{B}_1 \cap T_1 \cap TS_1 = \Gamma_1 | X = \Gamma_2, H^a)$ is therefore $\bar{b}_1 \times t_1 \times \gamma_1 \lambda(1-\tau)$.

Introducing the expressions for (a), (b), (c) and (d) into Eq. (8) produces

$$\frac{\Pr(Y_1 = \Gamma_1 | X = \Gamma_2, H^a)}{\Pr(Y_1 = \Gamma_1 | X = \Gamma_2, \bar{H}^a)} = \frac{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma'_1 b_1 \bar{t}_1}{\bar{b}_1 t'_1 \gamma_1 + \gamma'_1 b_1 \bar{t}'_1} \tag{14}$$

for ratio (i) in Eq. (7).

Again, it is the expression for (e) which is more complex. $\Pr(\bar{B}_2 \cap T_2 \cap TS_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_2, H^a)$ is the probability of the event that trace 2 was transferred during the struggle from an assailant having characteristic Γ_2, given that trace 1 has characteristic Γ_1, the suspect has characteristic Γ_2 and the suspect was an assailant who struggled with the victim on the crime scene. This probability is equal to

$$\bar{b}_2 \times t_2 \times \Pr(TS_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_2, H^a) \tag{15}$$

and the extension of the conversation to propositions F^a_2 and $\bar{F}^a_2$ to find Pr ($TS_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_2, H^a$) produces

$$\begin{aligned}&\Pr(TS_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_2, H^a) \\ &\quad = 1 \times \Pr(F^a_2 | Y_1 = \Gamma_1, X = \Gamma_2, H^a) + \gamma_2 \\ &\qquad \times \Pr(\bar{F}^a_2 | Y_1 = \Gamma_1, X = \Gamma_2, H^a)\end{aligned} \tag{16}$$

This development results from two possible explanations for the event $TS_2 = \Gamma_2$ (i.e., the event of the true source of a transferred trace in location 2 having characteristic Γ_2):

- either trace 2 was transferred from the suspect, in which case the suspect was the assailant who struggled with the victim in the location of trace 2 (proposition F^a_2);
- or trace 2 was transferred by another assailant (proposition $\bar{F}^a_2$), who has characteristic Γ_2 with a probability of γ_2.

The BN computes the probabilities $\Pr(F^a_2 | Y_1 = \Gamma_1, X = \Gamma_2, H^a)$ and $\Pr(\bar{F}^a_2 | Y_1 = \Gamma_1, X = \Gamma_2, H^a)$ by applying Bayes' theorem:

$$\begin{aligned}&\Pr(F^a_2 | Y_1 = \Gamma_1, X = \Gamma_2, H^a) \\ &\quad = \frac{\Pr(Y_1 = \Gamma_1 | F^a_2, X = \Gamma_2, H^a) \times \Pr(F^a_2 | H^a)}{\Pr(Y_1 = \Gamma_1 | X = \Gamma_2, H^a)}\end{aligned} \tag{17}$$

$$\begin{aligned}&\Pr(\bar{F}^a_2 | Y_1 = \Gamma_1, X = \Gamma_2, H^a) \\ &\quad = \frac{\Pr(Y_1 = \Gamma_1 | \bar{F}^a_2, X = \Gamma_2, H^a) \times \Pr(\bar{F}^a_2 | H^a)}{\Pr(Y_1 = \Gamma_1 | X = \Gamma_2, H^a)}\end{aligned} \tag{18}$$

These two ratios are made up of the following five probabilities:

- We develop $\Pr(Y_1 = \Gamma_1 | F^a_2, X = \Gamma_2, H^a)$ by extending the conversation over nodes B_1, T_1, and TS_1:

$$\begin{aligned}&\Pr(Y_1 = \Gamma_1 | F^a_2, X = \Gamma_2, H^a) \\ &\quad = 1 \times \Pr(\bar{B}_1 \cap T_1 \cap TS_1 = \Gamma_1 | F^a_2, X = \Gamma_2, H^a) \\ &\qquad + \gamma'_1 \times \Pr(B_1 \cap \bar{T}_1 | H^a) \\ &\quad = \bar{b}_1 t_1 \times \Pr(TS_1 = \Gamma_1 | F^a_2, X = \Gamma_2, H^a) + \gamma'_1 b_1 \bar{t}_1\end{aligned} \tag{19}$$

To find $\Pr(TS_1 = \Gamma_1 | F^a_2, X = \Gamma_2, H^a)$, we extend the conversation over propositions F^a_1 and $\bar{F}^a_1$:

$$\Pr(TS_1 = \Gamma_1 | F_2^a, X = \Gamma_2, H^a) = \gamma_1 \times \Pr(\bar{F}_1^a | F_2^a, H^a)$$

and to find $\Pr(\bar{F}_1^a | F_2^a, H^a)$, we apply Bayes' theorem:

$$\Pr(\bar{F}_1^a | F_2^a, H^a) = \frac{\overbrace{\Pr(F_2^a | \bar{F}_1^a, H^a)}^{1} \times \overbrace{\Pr(\bar{F}_1^a | H^a)}^{\lambda(1-\tau)}}{\underbrace{\Pr(F_2^a | H^a)}_{1-\lambda+\lambda(1-\tau)}} = \frac{\lambda(1-\tau)}{1-\lambda+\lambda(1-\tau)}$$

Inserting these results into Eq. (19) produces

$$\Pr(Y_1 = \Gamma_1 | F_2^a, X = \Gamma_2, H^a) = \bar{b}_1 t_1 \gamma_1 \left[\frac{\lambda(1-\tau)}{1-\lambda+\lambda(1-\tau)}\right] + \gamma_1' b_1 \bar{t}_1$$

- By definition,

$$\Pr(F_2^a | H^a) = 1 - \lambda + \lambda(1-\tau)$$

that is, given that the suspect was an assailant, the suspect was the assailant in location 2 either when there was only one assailant in both locations (probability $1 - \lambda$), or when there were two different assailants (probability λ), and the suspect struggled with the victim in location 2 (probability $1 - \tau$).

- Pr ($Y_1 = \Gamma_1 | X = \Gamma_2, H^a$) is the numerator of the first ratio in our LR (Eq. [14]):

$$\Pr(Y_1 = \Gamma_1 | X = \Gamma_2, H^a) = \bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1$$

- We develop $\Pr(Y_1 = \Gamma_1 | \bar{F}_2^a, X = \Gamma_2, H^a)$ by extending the conversation over nodes B_1, T_1, and TS_1:

$$\begin{aligned}\Pr(Y_1 &= \Gamma_1 | \bar{F}_2^a, X = \Gamma_2, H^a) \\ &= 1 \times \Pr(\bar{B}_1 \cap T_1 \cap TS_1 = \Gamma_1 | \bar{F}_2^a, X = \Gamma_2, H^a) \\ &\quad + \gamma_1' \times \Pr(B_1 \cap \bar{T}_1 | H^a) \\ &= \bar{b}_1 t_1 \times \Pr(TS_1 = \Gamma_1 | \bar{F}_2^a, X = \Gamma_2, H^a) + \gamma_1' b_1 \bar{t}_1\end{aligned} \tag{20}$$

To find $\Pr(TS_1 = \Gamma_1 | \bar{F}_2^a, X = \Gamma_2, H^a)$, we extend the conversation over propositions F_1^a and $\bar{F}_1^a$:

$$\Pr(TS_1 = \Gamma_1 | \bar{F}_2^a, X = \Gamma_2, H^a) = \gamma_1 \times \underbrace{\Pr(\bar{F}_1^a | \bar{F}_2^a, H^a)}_{0} = 0$$

Given that the suspect has characteristic Γ_2, the true source of a transferred trace in location 1 can only have characteristic Γ_1 if the assailant in location 1 was not the suspect (proposition $\bar{F}_1^a$). This unknown assailant possesses characteristic Γ_1 with a probability of γ_1. However, if the suspect was an assailant on the crime scene, then he must have been the assailant at either location 1 or location 2. According to our definitions of the propositions, it is therefore impossible that the suspect was not the assailant in location 1, given that he did not struggle with the victim in location 2, but was an assailant on the crime scene.

Inserting this result into Eq. (20) produces

$$\Pr(Y_1 = \Gamma_1 | \bar{F}_2^a, X = \Gamma_2, H^a) = \gamma_1' b_1 \bar{t}_1$$

If the suspect was an assailant, yet did not struggle with the victim in location 2, he must have been the assailant in location 1. However, because the trace in location 1 does not match the suspect's sample, trace 1 can only have characteristic Γ_1 if it is a background trace.

- And, by definition,

$$\Pr(\bar{F}_2^a | H^a) = \lambda\tau$$

as in Eq. (13).

Therefore, Eq. (17) is equal to

$$\begin{aligned}&\Pr(F_2^a | Y_1 = \Gamma_1, X = \Gamma_2, H^a) \\ &= \frac{\left\{\bar{b}_1 t_1 \gamma_1 \left[\frac{\lambda(1-\tau)}{1-\lambda+\lambda(1-\tau)}\right] + \gamma_1' b_1 \bar{t}_1\right\} \times [1 - \lambda + \lambda(1-\tau)]}{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1} \\ &= \frac{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1 [1 - \lambda + \lambda(1-\tau)]}{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1}\end{aligned}$$

and Eq. (18) to

$$\Pr(\bar{F}_2^a | Y_1 = \Gamma_1, X = \Gamma_2, H^a) = \frac{\gamma_1' b_1 \bar{t}_1 \lambda\tau}{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1}$$

Inserting these results in Eq. (16), and Eq. (16) into Eq. (15), gives us the probability of (e):

$$\begin{aligned}&\Pr(\bar{B}_2 \cap T_2 \cap TS_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_2, H^a) \\ &= \bar{b}_2 t_2 \left\{ \frac{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1 [1 - \lambda + \lambda(1-\tau)]}{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1} \right. \\ &\quad \left. + \gamma_2 \left[\frac{\gamma_1' b_1 \bar{t}_1 \lambda\tau}{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1}\right] \right\} \\ &= \frac{\bar{b}_2 t_2 \left\{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1 [1-\lambda+\lambda(1-\tau)] + \gamma_2(\gamma_1' b_1 \bar{t}_1 \lambda\tau)\right\}}{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1}\end{aligned}$$

Introducing the expressions for (e), (f), (g), and (h) into Eq. (8) produces

$$\frac{\Pr(Y_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_2, H^a)}{\Pr(Y_2 = \Gamma_2 | Y_1 = \Gamma_1, X = \Gamma_2, \bar{H}^a)} = \frac{\frac{\bar{b}_2 t_2 \left\{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1 [1-\lambda+\lambda(1-\tau)] + \gamma_2(\gamma_1' b_1 \bar{t}_1 \lambda\tau)\right\}}{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1} + \gamma_2' b_2 \bar{t}_2}{\bar{b}_2 t_2' \gamma_2 + \gamma_2' b_2 \bar{t}_2'}$$

for ratio (ii) in Eq. (7). Combining this result with Eq. (14), produces the following activity level LR for a case where the suspect's sample matches the second trace recovered on the crime scene:

$$\mathrm{LR} = \underbrace{\frac{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1}{\bar{b}_1 t_1' \gamma_1 + \gamma_1' b_1 \bar{t}_1'}}_{(i)} \times \underbrace{\frac{\frac{\bar{b}_2 t_2 \left\{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1 [1-\lambda+\lambda(1-\tau)] + \gamma_2(\gamma_1' b_1 \bar{t}_1 \lambda\tau)\right\}}{\bar{b}_1 t_1 \gamma_1 \lambda(1-\tau) + \gamma_1' b_1 \bar{t}_1} + \gamma_2' b_2 \bar{t}_2}{\bar{b}_2 t_2' \gamma_2 + \gamma_2' b_2 \bar{t}_2'}}_{(ii)}$$

Summary

The LR computed by the BN in Fig. 6*c* is the product of two ratios: one pertaining to the observation of the matching trace, and the other to the observation of the nonmatching trace. Whatever the order of these observations, the second observation is always conditional on the analytical result of the first trace observed.

If trace 1 matches the suspect's sample, the LR is:

$$\mathrm{LR} = \underbrace{\frac{\bar{b}_1 t_1[1-\lambda+\lambda\tau+\gamma_1\lambda(1-\tau)]+\gamma'_1 b_1\bar{t}_1}{\bar{b}_1 t'_1\gamma_1+\gamma'_1 b_1\bar{t}'_1}}_{\textit{matching trace}} \times \underbrace{\frac{\bar{b}_2 t_2\gamma_2\left\{\frac{(\bar{b}_1 t_1+\gamma'_1 b_1\bar{t}_1)\lambda\tau}{\bar{b}_1 t_1[1-\lambda+\lambda\tau+\gamma_1\lambda(1-\tau)]+\gamma'_1 b_1\bar{t}_1}\right\}+\gamma'_2 b_2\bar{t}_2}{\bar{b}_2 t'_2\gamma_2+\gamma'_2 b_2\bar{t}'_2}}_{\textit{non-matching trace given matching trace}} \tag{21}$$

and if trace 2 matches the suspect's sample, the LR is:

$$\mathrm{LR} = \underbrace{\frac{\bar{b}_1 t_1\gamma_1\lambda(1-\tau)+\gamma'_1 b_1\bar{t}_1}{\bar{b}_1 t'_1\gamma_1+\gamma'_1 b_1\bar{t}'_1}}_{\textit{non-matching trace}} \times \underbrace{\frac{\frac{\bar{b}_2 t_2\left\{\bar{b}_1 t_1\gamma_1\lambda(1-\tau)+\gamma'_1 b_1\bar{t}_1[1-\lambda+\lambda(1-\tau)]+\gamma_2(\gamma'_1 b_1\bar{t}_1\lambda\tau)\right\}}{\bar{b}_1 t_1\gamma_1\lambda(1-\tau)+\gamma'_1 b_1\bar{t}_1}+\gamma'_2 b_2\bar{t}_2}{\bar{b}_2 t'_2\gamma_2+\gamma'_2 b_2\bar{t}'_2}}_{\textit{matching trace given non-matching trace}} \tag{22}$$

Note that each ratio in these equations is an extension of the LR published for the one-trace transfer problem (Eq. [4]). In fact, the denominator and the second term in the numerator remain unchanged. The extension affects only the first term in the numerator, that is, the product $\bar{b}t$. This product is the probability that the recovered trace was transferred by an assailant during the alleged activity. It must be multiplied by a factor corresponding to the probability of the transferred trace's source having the observed characteristic. This factor will vary according to whether we consider one-trace (in Eq. [4] for the one-trace transfer problem, this factor is equal to 1) or two traces, or more accurately, whether we consider a case where there is the possibility of more than one assailant (6). In the latter case, it will further depend on whether we consider the matching trace or the nonmatching trace, and the first or the second of the two recovered traces. Note that Eqs (21) and (22) contain parameters which are actually prior probabilities. For this reason, the LR that is obtained is not to be intended in its usual sense because it appears that it no longer depends only upon the sample data.

To compare these expressions with each other and with Eq. (1), we multiply the two ratios, and rewrite them in the form of Eq. (1). Thus, Eq. (21) becomes:

$$\mathrm{LR} = \frac{\begin{array}{c}\bar{b}_2 t_2\gamma_2\left[(\bar{b}_1 t_1+b_1\gamma'_1\bar{t}_1)\lambda\tau\right]\\ +\gamma'_2 b_2\bar{t}_2\left\{\bar{b}_1 t_1[1-\lambda+\lambda\tau+\gamma_1\lambda(1-\tau)]+\gamma'_1 b_1\bar{t}_1\right\}\end{array}}{\left(\bar{b}_1 t'_1\gamma_1+\gamma'_1 b_1\bar{t}'_1\right)\times\left(\bar{b}_2 t'_2\gamma_2+\gamma'_2 b_2\bar{t}'_2\right)}$$

$$= \frac{\begin{array}{c}\bar{b}_1\bar{b}_2 t_1 t_2\lambda\tau\gamma_2+\bar{b}_1 b_2 t_1\bar{t}_2[1-\lambda+\lambda\tau+\lambda(1-\tau)\gamma_1]\gamma'_2\\ +b_1\bar{b}_2\bar{t}_1 t_2\lambda\tau\gamma'_1\gamma_2+b_1 b_2\bar{t}_1\bar{t}_2\gamma'_1\gamma'_2\end{array}}{\bar{b}_1\bar{b}_2 t'_1 t'_2\gamma_1\gamma_2+\bar{b}_1 b_2 t'_1\bar{t}'_2\gamma_1\gamma'_2+b_1\bar{b}_2\bar{t}'_1 t'_2\gamma'_1\gamma_2+b_1 b_2\bar{t}'_1\bar{t}'_2\gamma'_1\gamma'_2} \tag{23}$$

and Eq. (22):

$$\mathrm{LR} = \frac{\begin{array}{c}\bar{b}_2 t_2\{\bar{b}_1 t_1\gamma_1\lambda(1-\tau)+\gamma'_1 b_1\bar{t}_1[1-\lambda+\lambda(1-\tau)]\\ +\gamma_2(\gamma'_1 b_1\bar{t}_1\lambda\tau)\}+\gamma'_2 b_2\bar{t}_2[\bar{b}_1 t_1\gamma_1\lambda(1-\tau)+\gamma'_1 b_1\bar{t}_1]\end{array}}{\left(\bar{b}_1 t'_1\gamma_1+\gamma'_1 b_1\bar{t}'_1\right)\times\left(\bar{b}_2 t'_2\gamma_2+\gamma'_2 b_2\bar{t}'_2\right)}$$

$$= \frac{\begin{array}{c}\bar{b}_1\bar{b}_2 t_1 t_2\lambda(1-\tau)\gamma_1+\bar{b}_1 b_2 t_1\bar{t}_2\lambda(1-\tau)\gamma_1\gamma'_2\\ +b_1\bar{b}_2\bar{t}_1 t_2[1-\lambda+\lambda(1-\tau)+\lambda\tau\gamma_2]\gamma'_1+b_1 b_2\bar{t}_1\bar{t}_2\gamma'_1\gamma'_2\end{array}}{\bar{b}_1\bar{b}_2 t'_1 t'_2\gamma_1\gamma_2+\bar{b}_1 b_2 t'_1\bar{t}'_2\gamma_1\gamma'_2+b_1\bar{b}_2\bar{t}'_1 t'_2\gamma'_1\gamma_2+b_1 b_2\bar{t}'_1\bar{t}'_2\gamma'_1\gamma'_2} \tag{24}$$

These two equations are identical for $\tau = 0.5$. In this case, we do not need to differentiate between trace 1 and trace 2, and only need to distinguish between the matching trace $i \in \{1, 2\}$ with characteristic Γ_i, and the nonmatching trace $j \in \{1, 2\}$, $j \neq i$, with characteristic Γ_j:

$$\mathrm{LR} = \frac{\begin{array}{c}\frac{1}{2}\bar{b}_i\bar{b}_j t_i t_j\lambda\gamma_j+\bar{b}_i b_j t_i\bar{t}_j[1-\lambda+\frac{1}{2}\lambda+\frac{1}{2}\lambda\gamma_i]\gamma'_j\\ +\frac{1}{2}b_i\bar{b}_j\bar{t}_i t_j\lambda\gamma'_i\gamma_j+b_i b_j\bar{t}_i\bar{t}_j\gamma'_i\gamma'_j\end{array}}{\bar{b}_i\bar{b}_j t'_i t'_j\gamma_i\gamma_j+\bar{b}_i b_j t'_i\bar{t}'_j\gamma_i\gamma'_j+b_i\bar{b}_j\bar{t}'_i t'_j\gamma'_i\gamma_j+b_i b_j\bar{t}'_i\bar{t}'_j\gamma'_i\gamma'_j} \tag{25}$$

The numerator and denominator of this equation each consist of the four possible combinations of background and transferred traces for the matching and the nonmatching trace (i.e., both traces were transferred, only the matching trace was transferred, only the nonmatching trace was transferred, and both traces are background traces). In the denominator and in the fourth term of the numerator, the probabilities of the observations consist of the product of the corresponding background, transfer, and match probabilities. These describe events that are independent of the suspect's characteristic and of the suspect's possible involvement in the struggle with the victim. The first three terms of the numerator, however, are not independent of the suspect's involvement. These contain the additional probabilities of λ and τ (which, in Eq. [25] is equal to $\frac{1}{2}$), in addition to the background, transfer, and match probabilities. More specifically:

- The first term in the numerator considers the event that both traces were transferred during the struggle. Apart from the appropriate background and transfer probabilities, the probability of observing one matching and one nonmatching trace, given that the suspect was an assailant, is equal to the match probability of the nonmatching trace's characteristic (i.e., γ_j) times the probability that there were two different assailants (i.e., λ) times the probability that the suspect was the assailant in location 1 (in this case, $\frac{1}{2}$).
- The second term in the numerator describes the event that only the matching trace was transferred during the struggle. In this case, there are three possibilities: the suspect could have been the assailant in both locations (probability of $1 - \lambda$), the suspect

could have been the assailant only in the location of the matching trace (here, probability of $\frac{1}{2}\lambda$), or the suspect could have been the assailant only in the location of the nonmatching trace (here, probability of $\frac{1}{2}\lambda$). In the last case, the assailant in the other location also had the matching characteristic with a probability of γ_j. The sum of the probabilities of each of these possible events makes up the additional factor in the square brackets in Eq. (25).

- The third term in the numerator describes the event that only the nonmatching trace was transferred. As the suspect could not have transferred the nonmatching trace, this is only possible if the suspect was the assailant at the location of the matching trace (here, probability of $\frac{1}{2}\lambda$).

The LR computed by the BN thus combines the probabilities defined in this paper in a logical way.

Discussion

In this section, we analyze Eqs (23)–(25). First, we compare these expressions to Eq. (1); then, we show how Eq. (25) may reduce to both the source level LR for two traces and the activity level LR for a single trace under the appropriate assumptions; and finally, we discuss the extension of the model to n traces.

Comparison with Eq. (1)

Rewriting Eq. (1) with the subscript i for the probabilities referring to the matching trace, and the subscript j for the probabilities referring to the nonmatching trace, produces (6):

$$\mathrm{LR} = \frac{\frac{1}{2}\bar{b}_i\bar{b}_j t_i t_j(1-2q)\gamma_j + \frac{1}{2}\bar{b}_i b_j t_i \bar{t}_j(1+\gamma_i)\gamma'_j + \frac{1}{2}b_i\bar{b}_j\bar{t}_i t_j\gamma'_i\gamma_j + b_i b_j \bar{t}_i\bar{t}_j\gamma'_i\gamma'_j}{\bar{b}_i\bar{b}_j t_i t_j(1-2q)\gamma_i\gamma_j + \bar{b}_i b_j t_i\bar{t}_j\gamma_i\gamma'_j + b_i\bar{b}_j\bar{t}_i t_j\gamma'_i\gamma_j + b_i b_j\bar{t}_i\bar{t}_j\gamma'_i\gamma'_j} \tag{26}$$

A comparison of this equation with Eq. (25) shows that this equation assumes $\tau = 0.5$, and uses the variable q whereas we have used the variable λ. The differences are due to the different definitions underlying q and λ. This can be seen by setting $q = 0$ and $\lambda = 1$: in this case, these variables disappear from the equations and the two LRs become identical.

The difference between q and λ is that the definition of q is limited to the event of two transferred traces, whereas λ is defined at the level of the propositions, independently of whether the traces were transferred during the struggle. More specifically, expression $1 - 2q$ (in Eq. [26]) denotes the probability that two transferred traces come from different assailants (6). It therefore only applies to the first term in the numerator and the first term in the denominator, where the probabilities describe the event of two transferred traces. Probability λ (in Eq. [25]), on the other hand, describes the prior probability that the assailant in location 1 was not the same assailant as the assailant in location 2. This definition is not limited to only the transferred traces, and therefore appears in the first, second, and third terms of the LR's numerator (Eq. [25]). Note that λ does not figure in the LR's denominator, because the denominator considers the observations of the two traces independently of each other.

Owing to these different definitions:

- the additional factor of $1 - 2q$ in the first term of the denominator of Eq. (26) makes this denominator smaller than the denominator of Eq. (25);
- the additional factor of $1 - \lambda$ in the second term of the numerator in Eq. (25) makes this term greater in Eq. (25) than in Eq. (26); and
- probability λ in the third term of the numerator in Eq. (25) makes this term smaller in Eq. (25) than in Eq. (26).

Numerically, the impact of these differences depends on the values assigned to the background and transfer probabilities. (That is, in most cases Eq. [26] will produce a slightly greater LR, yet if large values are assigned to b_i and t_j, and small values to b_j and t_i, Eq. [25] may produce the greater LR owing to the greater impact of the numerator's third term.)

Verification of the Model

To verify the results produced by our model, we show that Eq. (25) reduces to the source level LR for two traces and to the activity level LR for a single trace when assumptions are made to simulate these two situations.

First, we observe that letting the background (b_i and b_j) and transfer probabilities (t_i, t'_i, t_j, and t'_j) tend toward 1 or 0 leads to the expected, logical results: the computed LR is an increasing function of t_i and a decreasing function of t_j. If $t_i = t'_i = t_j = t'_j = 0$, it reduces to 1, and when $t_i = t'_i = t_j = t'_j = 1$ and $\bar{b}_i = \bar{b}_j = 1$, it tends toward the source level LR, which is $\frac{1}{2\gamma_i}$ for $\lambda = 1$ and $\tau = \frac{1}{2}$ (3):

$$LR = \frac{\frac{1}{2}\gamma_j}{\gamma_i\gamma_j} = \frac{1}{2\gamma_i}$$

Second, the computations of the BN can also be reduced to the activity level LR for a single trace (Eq. [4]). To do this, we consider a scenario in which *another* assailant, matching trace j, has already been found, and one assumes that if trace j was transferred by one of the assailants, then it was transferred by this other assailant. We extended Fig. 6*c* to illustrate this new situation by adding a node Z for the second suspect, and a node H^Z for a second pair of general propositions, pertaining to this second suspect (Fig. 7). The LR deduced from this model with respect to the first suspect (see Appendix for the derivation) is now:

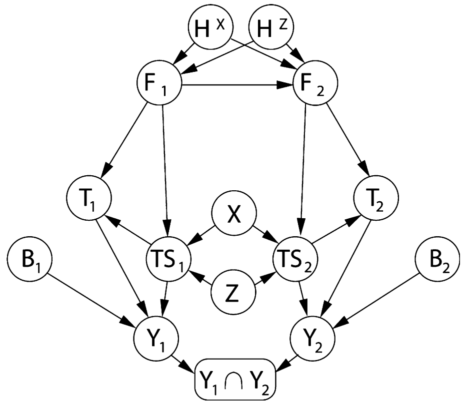

FIG. 7—*BN extended to include a second suspect, denoted Z, in a case where there were two different assailants. A node* H^Z *was added containing the same propositions as in* H *(now renamed* H^X*), but for this second suspect.*

$$\mathrm{LR} = \frac{\bar{b}_i t_i + \gamma'_i b_i \bar{t}_i}{\bar{b}_i t'_i \gamma_i + \gamma'_i b_i \bar{t}'_i}$$

which corresponds to the activity level LR (Eq. [4]) for a single trace (5).

Extension to n Traces

In an *n-trace* transfer problem, we consider *n* different traces recovered on the crime scene to come from *n* distinct sources. The organized structure of the BN in Fig. 6*c* easily lends it to an extension to any number of traces by transforming it into an *object-oriented BN* (OOBN). An OOBN allows the user to evaluate more complex problems by combining different objects in a hierarchical structure (16). An object may be a simple random variable (like the nodes in a regular BN), or a separate, complex model, such as another BN (29). Thus, the main advantage of an OOBN is its capacity to differentiate between several hierarchical levels and to combine a set of nodes from different models. This is particularly useful for combining a set of identical network fragments that form a repetitive pattern in a regular BN (15,19).

In the case of the two-trace problem, one can represent each group of variables specific to one-trace as a separate object. In this extension of the model, we assume that $\lambda = 1$ (i.e., there are *n* different perpetrators for a case with *n* traces) and omit node *L* in the BN. Thus, the two-trace problem in Fig. 6*c* becomes an OOBN with only four objects: two random variables and two subnetworks (Fig. 8*a*). The two network fragments hidden in the interface nodes of *trace 1* and *trace 2* in Fig. 8*a* are shown in Fig. 9*a,b*, respectively. This OOBN has the same structure as the BN in Fig. 6*c* (without node *L*), only decomposed into three separate elements.

The OOBN structure for two traces suggests a logical way to extend the model to additional traces. New traces are added in the same way trace 2 was added to trace 1: the observation of each new trace's characteristic depends on the general variables *H* and *X*, and on the specific hypotheses (contained in nodes of type *F*) of each of the previously observed traces (Figs 8*b* and 9*c*). By this means, one can construct a general model for *m* different traces ($m \geq n$), and then designate, through an additional node *N*, the number of different traces *n* for which one would like the BN to compute an LR.

The program *Hugin Researcher* is only limited by the amount of memory it can use. This limit lies at 4GB, which is great enough to allow for hundreds of traces to be modeled with this OOBN (the exact number of traces will depend on the number of analytical traits defined as the states of nodes *X*, *Y*, and *TS*).

The additional variable *N* also allows the user to introduce an uncertainty on the number of sources if this is not clearly defined by the circumstantial information of the case. This OOBN clearly describes the dependencies assumed among the variables, and rigorously applies the laws of probability to compute the LR of interest.

TABLE 10—*Definitions of the additional propositions that figure in the BN shown in Fig. 7.*

Nodes	States	Definitions of the States
H^X	H^a_X	Suspect 1 was engaged in a struggle with the victim
	$\bar{H}^a_X$	Suspect 1 was not engaged in a struggle with the victim
H^Z	H^a_Z	Suspect 2 was engaged in a struggle with the victim
	H^a_Z	Suspect 2 was not engaged in a struggle with the victim
F_i	$F^a_{i,X}$	Suspect 1 was engaged in a struggle with the victim in the location of trace *i*
	$F^a_{i,Z}$	Suspect 2 was engaged in a struggle with the victim in the location of trace *i*
	$\bar{F}^a_i$	Neither suspect 1, nor suspect 2, was not engaged in a struggle with the victim in the location of trace *i*
$F_{j \neq i}$	$F^a_{j \neq i,X}$	Suspect 1 was engaged in a struggle with the victim in the location of trace *j*
	$F^a_{j \neq i,Z}$	Suspect 2 was engaged in a struggle with the victim in the location of trace *j*
	$\bar{F}^a_{j \neq i}$	Neither suspect 1, nor suspect 2, was not engaged in a struggle with the victim in the location of trace *j*

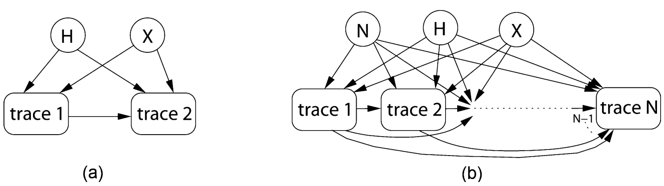

FIG. 8—*Object-oriented BNs for (a) two traces, and (b) n traces. In (b), the additional node* N *allows the user to specify the number of different traces the BN should consider in the evaluation. Here, the dotted line represents traces 3 to N-1. Each additional trace has an incoming arrow from nodes* N, H, *and* X, *and from each of the previously observed traces.*

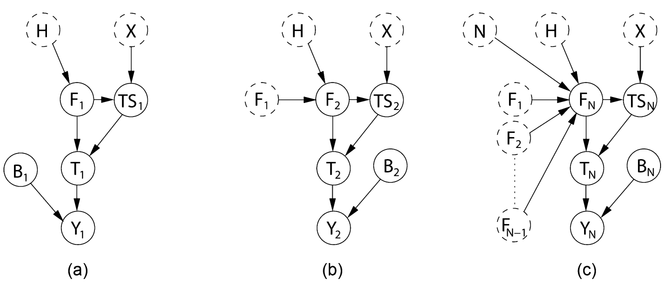

FIG. 9—*The network fragments hidden in the interface nodes of (a) trace 1, (b) trace 2, and (c) trace N in Fig. 8. The nodes with a dashed contour are nodes figuring either in the master network in Fig. 8, or in a different network fragment. The dotted line in (c) represents nodes* F_3 *to* F_{N-2}. *These are all a parent to node* F_N.

Conclusions

Forensic scientists are faced with the need of addressing increasingly complex inference problems for assessing the value of scientific evidence. Two-trace problems are a typical example for this. They are a realistic problem which, up to now, forensic statisticians have addressed with an algebraic approach for calculating LRs. These applications have led to efficient results for simple evidential assessments, yet quickly lead to mathematically sophisticated expressions when applied to more complex problems. For an increasing number of variables and an increasing number of conditional probabilistic relationships between these variables, purely theoretical developments make it difficult to maintain a transparent and error-free approach. The algebraic approach thus reaches its limits when it is applied to increasingly complex inference problems.

The aim of this study was to investigate a new way for computing LRs, a graphical approach based on the construction of BNs. These graphical models overcome the hurdle of complexity by:

- decomposing complicated events into a set of distinct variables;
- describing and visualizing the assumed dependencies among the variables;

- rigorously handling probabilistic calculations in a mathematically robust environment;
- easily incorporating additional variables into existing models; and
- coherently combining and structuring different aspects of a problem as separate objects in distinct hierarchical levels of an OOBN.

Thus, the construction of BNs provides a transparent approach to inference problems that is not limited by an increasing number of variables and probabilistic relationships, and not prone to careless mathematical errors that may occur when using or developing an algebraic formula.

In the context of the two-trace transfer problem, the development of a BN demonstrated the potential of such graphical probability models by producing a new activity level LR that relaxes assumptions made in previous algebraic developments. In addition, the graphical structure readily presents itself to extensions to more complex problems such as the *n*-trace problem at the activity level. Thus, the development of BNs allows forensic scientists to progress in the field of evidential interpretation by providing a tool to tackle more complex inference problems in a structured and logical way.

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Appendix: Derivation for the Activity Level LR for Fig. 7

The aim of this derivation is to show that the introduction of a second suspect (denoted *Z*) matching trace *j* allows one to reduce the activity level LR for two traces to the activity level LR for a single trace. For this evaluation, we assume that $\lambda = 1$, that is, that there were two different assailants. Further, we assume that the second suspect was engaged in a struggle with the victim in the location of trace *j* (i.e., proposition $F^a_{j,Z}$). To differentiate the propositions referring to suspect 1 from those referring to suspect 2, the former now contain an additional subscript *X*, and the latter a subscript *Z* (see Table 10 for the definitions of these different propositions). We want to obtain the LR for suspect *X*. With respect to suspect *X*, we shall assume that we observe first the nonmatching trace *j*, and then the matching trace *i*. The LR is computed for propositions H^a_X and $\bar{H}^a_X$, given proposition $F^a_{j\neq i,Z}$, denoted in the following developments as $F^a_{j,Z}$:

$$LR = \frac{\Pr(Y_j|X, Z, F^a_{j,Z}, H^a_X)}{\Pr(Y_j|X, Z, F^a_{j,Z}, H^a_X)} \times \frac{\Pr(Y_i|Y_j, X, Z, F^a_{j,Z}, H^a_X)}{\Pr(Y_i|Y_j, X, Z, F^a_{j,Z}, \bar{H}^a_X)} \qquad (27)$$

We define the observations as: $Y_j = \Gamma_j$, $Y_i = \Gamma_i$, $Z = \Gamma_j$ and $X = \Gamma_i$.

For the numerator of the first ratio, the extension of the conversation over variables B_j, T_j, and TS_j produces:

$$\begin{aligned}&\Pr(Y_j|X,Z,F^a_{j,Z},H^a_X)\\&\quad=1\times\Pr(\bar{B}_j\cap T_j\cap TS_j=\Gamma_j|Z,F^a_{j,Z},H^a_X)+\gamma'_j\times\Pr(B_j\cap\bar{T}_j|H^a_X)\\&\quad=\bar{b}_jt_j\times\Pr(TS_j=\Gamma_j|Z,F^a_{j,Z})+\gamma'_jb_j\bar{t}_j\end{aligned}$$

Given $F^a_{j,Z}$ and $Z = \Gamma_j$,

$$\Pr(TS_j=\Gamma_j|Z,F^a_{j,Z})=1$$

such that

$$\Pr(Y_j|X,Z,F^a_{j,Z},H^a_X)=\bar{b}_jt_j+\gamma'_jb_j\bar{t}_j$$

For the denominator of the first ratio, we observe the same development:

$$\begin{aligned}&\Pr(Y_j|X,Z,F^a_{j,Z},\bar{H}^a_X)\\&\quad=1\times\Pr(\bar{B}_j\cap T_j\cap TS_j=\Gamma_j|Z,F^a_{j,Z},\bar{H}^a_X)+\gamma'_j\times\Pr(B_j\cap\bar{T}_j|\bar{H}^a_X)\\&\quad=\bar{b}_jt_j\times\Pr(TS_j=\Gamma_j|Z,F^a_{j,Z})+\gamma'_jb_j\bar{t}_j\end{aligned}$$

with

$$\Pr(TS_j=\Gamma_j|Z,F^a_{j,Z})=1$$

such that

$$\Pr(Y_j|X,Z,F^a_{j,Z},\bar{H}^a_X)=\bar{b}_jt_j+\gamma'_jb_j\bar{t}_j$$

Given $F^a_{j,Z}$ and $Z = \Gamma_j$, the event $Y_j = \Gamma_j$ is independent of the propositions pertaining to suspect 1 (H^a_X and $\bar{H}^a_X$). This makes the first ratio in Eq. (27) equal to 1.

The extension of the conversation for the numerator of the second ratio produces:

$$\begin{aligned}&\Pr(Y_i|Y_j,X,Z,F^a_{j,Z},H^a_X)\\&\quad=1\times\Pr(\bar{B}_i\cap T_i\cap TS_i=\Gamma_i|X,F^a_{j,Z},H^a_X)+\gamma'_i\times\Pr(B_i\cap\bar{T}_i|H^a_X)\\&\quad=\bar{b}_it_i\times\Pr(TS_i=\Gamma_i|X,F^a_{j,Z},H^a_X)+\gamma'_ib_i\bar{t}_i\end{aligned}$$

Given that suspect 1 was one of the assailants (proposition H^a_X), and that suspect 2 was the assailant in the location of trace j (proposition $F^a_{j\neq i,Z}$), suspect 1 must have been the assailant in the location of trace i (proposition $F^a_{i,X}$). Therefore,

$$\Pr(TS_i=\Gamma_i|X,F^a_{j,Z},H^a_X)=\Pr(TS_i=\Gamma_i|X,F^a_{i,X})=1$$

and

$$\Pr(Y_i|Y_j,X,Z,F^a_{j,Z},H^a_X)=\bar{b}_it_i+\gamma'_ib_i\bar{t}_i$$

For the denominator of the second ratio, we obtain:

$$\begin{aligned}&\Pr(Y_i|Y_j,X,Z,F^a_{j,Z},\bar{H}^a_X)\\&\quad=1\times\Pr(\bar{B}_i\cap T_i\cap TS_i=\Gamma_i|\bar{H}^a_X)+\gamma'_i\times\Pr(B_i\cap\bar{T}_i|\bar{H}^a_X)\\&\quad=\bar{b}_it'_i\times\Pr(TS_i=\Gamma_i|\bar{H}^a_X)+\gamma'_ib_i\bar{t}'_i\end{aligned}$$

Under $\bar{H}^a_X$, suspect 1 was not one of the assailants, and trace i must have been transferred by an unknown assailant if it was transferred during the assault. This other assailant has characteristic Γ_i with a probability of γ_i:

$$\Pr(TS_i=\Gamma_i|\bar{H}^a_X)=\gamma_i$$

and

$$\Pr(Y_i|Y_j,X,Z,F^a_{j,Z},\bar{H}^a_X)=\bar{b}_it'_i\gamma_i+\gamma'_ib_i\bar{t}'_i$$

Therefore, Eq. (27) is equal to:

$$\begin{aligned}LR&=\frac{\bar{b}_jt_j+\gamma'_jb_j\bar{t}_j}{\bar{b}_jt_j+\gamma'_jb_j\bar{t}_j}\times\frac{\bar{b}_it_i+\gamma'_ib_i\bar{t}_i}{\bar{b}_it_i\gamma_i+\gamma'_ib_i\bar{t}_i}\\&=1\times\frac{\bar{b}_it_i+\gamma'_ib_i\bar{t}_i}{\bar{b}_it_i\gamma_i+\gamma'_ib_i\bar{t}_i}\\&=\frac{\bar{b}_it_i+\gamma'_ib_i\bar{t}_i}{\bar{b}_it_i\gamma_i+\gamma'_ib_i\bar{t}_i}\end{aligned}$$

This LR is equal to the activity level LR for a single trace (Eq. [4]).